

Scale invariance in cosmology and physics

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The fundamental laws of physics are required to be invariant under *local* spatial scale change. In 3-dimensional space, scale invariance inevitably leads to a variation in two physical constants – the Planck constant \hbar and speed of light c – as specific functions of the local scale. *The values of \hbar and c measured at a given spacetime point are dependent on the spatial scale in the vicinity of that point.* Specifically, they vary as $\hbar \propto a^{\frac{1}{2}}$ and $c \propto a^{-\frac{1}{2}}$ with a being the local scale factor. One direct consequence is that the expanding universe progressively alters the values of these physical constants which in turn affect the evolution of the universe itself. Friedmann’s equations violate scale invariance and neglect to account for the scale dependency of the constants specified above. We build a cosmological model which is fully consistent with scale invariance *and* which, at the same time, respects Lorentz invariance; as a result, all existing fundamental physical laws remain intact in the expanding universe. This model – *rigorously* derived from Einstein’s field equation – leads to a universe different from the ones depicted in Friedmann’s standard model.

We apply our model to resolve a series of observational and theoretical difficulties encountered in modern cosmology: the “runaway density parameter” problem, the budgetary shortfall, several cosmic coincidences, the oldness problem, the horizon problem, the age problem, the past and destiny of the universe. Our model does not resort to fine tuning or the inflationary universe hypothesis. A modification to the Hubble law and Hubble constant, and a new interpretation of the brightness-redshift relationship empirically observed in Type Ia supernovae are direct consequences of our model: (a) *The Hubble constant has been inadvertently overestimated by a factor of 9/5*; so has the critical density ρ_c by $(9/5)^2$; (b) Our new photometric distance-redshift relationship $d_L = \frac{2c}{3H_0}(1+z)^{\frac{3}{2}} \ln(1+z)$ with only one free parameter $H_0 = 37$ fits to the high- z objects as equally well as the traditional relationship does with three parameters $H_0 = 70.5$, $\Omega_M = 0.27$, $\Omega_\Lambda = 0.73$.

These facts help us draw two conclusions: (i) With $H_0 = 37$, the critical density ρ_c is only 0.28 time the value previously thought; the matter content is *solely* responsible for the flatness of space; *dark energy is absent*. (ii) The corrected Hubble value $H_0 = 37$ restores the age estimate back to 17.6 *Gy* (via the universal age formula $t_0 = 2/3H_0^{-1}$). In addition, they raise the possibility that the universe expansion is not accelerating, but rather a result of the physical laws of Nature – including c and \hbar – progressively adapting to new spatial scale as the universe expands. Finally, we discuss an array of implications of scale invariance in the larger context of physics.

I. MOTIVATION

The foundation of modern cosmology is based largely on Friedmann’s equations which describe the evolution of the universe [1]. Friedmann’s equations predict that the universe is dynamic, meaning that its spatial scale factor varies along the cosmic time. On the observational front, it has been established beyond doubt that the universe has been expanding [2]. While this is a triumph of theoretical cosmology, Friedman’s equations are internally inconsistent. Their inconsistency occurs at two levels:

- At the conceptual level, as the universe evolves, physical laws – including Friedmann’s equations – must adjust themselves to the new spatial scale. This element of *adaptedness* is currently missing from Friedmann’s theory and, as a matter of fact, from every law of physics. This issue is both logically and practically important because the adaptation of Friedmann’s equations, among other laws, to the scale factor will necessarily alter the subse-

quent evolution of the scale factor itself.

- At the technical level, consider Friedmann’s equation:

$$\frac{1}{a^2} \left(\frac{da}{dt} \right)^2 = \frac{8\pi G}{3} \rho - \frac{kc^2}{a^2} \quad (1)$$

where a and k are the scale factor and the spatial curvature, ρ the mass density, G and c the gravitational constant and speed of light. Although all three terms in the equation are of the same unit, sec^{-2} , they scale differently with respect to a : $\frac{8\pi G}{3} \rho \propto a^{-3}$, $\frac{kc^2}{a^2} \propto a^{-2}$, and how $\frac{1}{a^2} \left(\frac{da}{dt} \right)^2$ scales depends on k (i.e., open/flat/closed universes evolve differently). Although this state of affairs is conventionally seen as desirable (since it “richly” generates different scenarios for the universe evolution), the mismatch in scaling is the root of a host of problems and difficulties that the Big Bang theory encountered and must be remedied by the inflationary universe hypothesis and the cosmological constant. As we shall show, these difficulties will *disappear* once the impaired scaling in Friedmann’s equations is corrected.

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The goal of our undertaking is two-fold: (I) We build a cosmological model that adjusts in accordance with the scale change as the universe evolves. At the same time, our model will respect Lorentz invariance. The end result is a new set of equations to replace Friedmann’s equations. (II) We generalize the fundamental laws of physics to reflect their adaptation to the scale variation. The generalized laws will suitably describe physics in the evolutionary universe. The key to accomplishing these tasks is found in the scale invariance consideration.

Our report is structured as follows. We shall elaborate on the impact of scale invariance requirement on the laws of physics in Section II. Scaling rules of the Planck constant \hbar and speed of light c are derived. Section III contains a new cosmological model: we generalize the Robertson-Walker metric to incorporate the scaling rule for c , and rigorously derive from Einstein’s field equation a new set of evolution equations – which we shall call the modified Friedmann equations. Section IV provides a detailed exposition on the compatibility of scale-dependent light speed with Lorentz invariance. Section V describes our modifications to Lemaitre’s redshift formula, to the Hubble law and Hubble constant, and to the luminosity-redshift relationship. We then apply them to resolve the age problem and the “dark energy” problem. Based on these modifications, we next present a critical analysis of Type Ia supernovae and offer a tentative interpretation in place of the mainstream “accelerating” universe explanation. Section VI applies our model to cure several theoretical difficulties encountered in the Big Bang theory. Section VII discusses the implications of our model in the context of physics. The last two sections compare our approach with existing literature and conclude the report.

II. SCALE INVARIANCE PRINCIPLE AND SCALING PROPERTIES OF THE FUNDAMENTAL CONSTANTS

A. Invitation: A gedanken experiment

We approach this problem by asking ourselves the following question: *How do physical laws appear in a region of space that has undergone an expansion?*

Suppose that a physicist (Alice) is conducting a series of experiments in her lab. Alice, say, wants to test a number of predictions of quantum mechanics. In particular, she wants to test two of its theoretical predictions: the formula for Bohr’s radius, $r_{Bohr} = \hbar^2/(m_e e^2)$, and the formula for the energy levels of the hydrogen atom, $E_n = -m_e e^4/(2\hbar^2 n^2)$. At her disposal, she measures: (i) the size of the hydrogen atom; (ii) the frequency (thus wavelength and energy) of the photon emitted from atom. Let us say that all her measurements so far confirm the formulae of Bohr’s radius and of the energy levels. Now imagine that the space that encloses her lab surreptitiously scales up by a factor of 2. Would Alice – using

the apparatus in her lab – be able to detect such an expansion? How would her measurements of Bohr’s radius and the energy levels appear if she is to repeat her experiments?

Our intuition impels us to expect that Alice would not be able to tell apart her new situation. Since every length in her lab gets scaled up, she would find the ratio between the photon’s wavelength and the atom’s size to be the same as before. If she sends back the photon to the atom, the atom will happily re-absorb the photon. All appear proper to Alice; she would have no reason to suspect anything amiss. In her observation, everything is consistent with one another and is compatible with the predictions of quantum mechanics.

Next, imagine that a spectator (Bob) lives far apart from Alice’s lab. His living space, however, does not scale up as hers does. By observation, he would be able to tell the doubling in size of her lab. (Or his space gets halved, but that point is not crucial.) Upon learning of the space enlargement from him, Alice has two options:

- (A) Ignore his advice: as long as she retains her “stretched” ruler (and “slowed” clock, as we shall see shortly), every test in her lab would be consistent with one another and with quantum theory’s predictions.
- (B) Ask him to send in *his* ruler and clock: using these new tools, she would find a doubling in both the atom’s size and the photon’s wavelength. Accordingly, in Bob’s units of length and time, the photon’s energy is halved (since $E = \hbar c/\lambda$). (The frequency of the photon, curiously, is *not* halved; rather, it gets reduced by a factor of $2\sqrt{2}$ for a reason to be explained momentarily.) There arises an issue: The fundamental constants must vary such that Bohr’s radius, $r_{Bohr} = \hbar^2/(m_e e^2)$, doubles to agree with the length measurement, and the energy levels, $E_n = -m_e e^4/(2\hbar^2 n^2)$, get halved so that the atom can happily re-absorb the photon.

More precisely, in option (B), to accommodate her new situation in which the surrounding of her lab has undergone a scale factor a , Alice would need to adjust the Schrödinger equation describing the hydrogen atom *in her lab*

$$i\hbar \frac{\partial}{\partial t} \Psi = \left[-\frac{\hbar^2}{2m_e} \nabla^2 - \frac{e^2}{r} \right] \Psi \quad (2)$$

accordingly. If she interprets e and m_e as intrinsic properties of the electron which should remain unaffected by the scale change, the only logical step for her is to rescale \hbar and t . Under the spatial scaling $\vec{x} \rightarrow a\vec{x}$, the rescaling rules should be

$$\hbar \rightarrow a^{\frac{1}{2}} \hbar \quad (3)$$

$$t \rightarrow a^{\frac{3}{2}} t \quad (4)$$

The rescaling of time t means that if $a > 1$ the long hand of her clock requires more time to cover a full revolution

than before. Take $a = 2$, the “slowing down” effect in clocks is $2\sqrt{2}$ which explains the unintuitive reduction for frequency alluded previously. That is to say, the frequency scales as:

$$\nu \rightarrow a^{-\frac{3}{2}}\nu \quad (5)$$

Let us recap the situation: A change in reference scale inevitably leads to changes in \hbar and in the clock rate. A doubling in scale incurs a doubling in \hbar and a $2\sqrt{2}$ -time increase in the clock rate. This leads to a $\sqrt{2}$ -time reduction in velocity, including the velocity of light!

One might object that these conclusions are a trivial exercise of dimensional analysis, and that since \hbar and c are not dimensionless, one can change their apparent value to any desirable value by changing the units (e.g., see [8]). The latter objection is not valid in the following situation: Imagine that Bob now also conducts his own experiments then compares his results with Alice’s lab record. There is *one and only one* set of units available for Bob. He must make sure that her lab record is in accordance with *his* ruler and clock. If Alice has opted to stick with her ruler and clock, Bob – in full awareness of the space expansion effect on her measurements – must adjust \hbar in her record accordingly before doing a comparison with his own record.

The *relative* variation in \hbar (and also c) is thus measurable, and is a *meaningful* concept. This is a crucial point that makes our gedanken experiment relevant to reality.

B. Relevance of scale change in cosmology

What does our gedanken situation have to do to reality? The answer is that its most direct relevance is in cosmology. As we gaze deeper into the night sky, not only do we observe a galaxy – and one of its hydrogen atoms – at their youth, more importantly, we observe a galaxy and an atom that existed at a smaller scale, *a scale at which \hbar is smaller and c is higher*. If we are to compare the spectrum of an ancient hydrogen atom with that of an hydrogen atom in our now-enlarged universe, we have *no* choice but adjust the Schrödinger equation accordingly. In particular, we must use a smaller Planck constant for the distant hydrogen atom!

Consider an array of galaxies that line up along one direction of view from Earth. The farther away the galaxy, the smaller the value of \hbar and the higher the value of c that the galaxy experienced. Unlike Alice the physicist who could afford to ignore Bob’s advice and would run into no trouble by sticking with her “corrupted” ruler and clock, we – in the role of Bob the spectator – must progressively adjust \hbar and c to describe the farther and farther galaxies. *This effect has been overlooked in the standard paradigm of cosmology.*

To appreciate the significance of this observation, let us rephrase the situation: The information encoded in the photon emitted from the hydrogen atom at a distant

galaxy reflects the properties *of the atom itself* which corresponded to a smaller Planck constant! This is in radical contrast to the conventional interpretation in cosmology which simply holds that the photon gets redshifted in transit from the galaxy to Earth. The difference is stark: if the universe has doubled in size since the time the photon left the galaxy (i.e., $a = 2$), the conventional “redshift” explanation would have the photon frequency to halve, whereas our rescaling rules would predict a $2\sqrt{2}$ -time reduction in the photon frequency! This crucial difference will have sweeping ramifications in cosmography, a topics which we will investigate in great details in Section V.

Where does the additional $\sqrt{2}$ reduction come from? Answer: It comes from the increase in \hbar (per Eq(3)), or equivalently, from $E = \hbar\nu$ in which $E \propto a^{-1}$.

The additional reduction in frequency can also be obtained in another way: the Heisenberg uncertainty principle, $[\hat{x}, \hat{p}] = i\hbar$ leads to the following scaling rule for momentum and, subsequently, to that for velocity (recalling that mass is unchanged):

$$p \rightarrow a^{-\frac{1}{2}}p \quad (6)$$

$$v \rightarrow a^{-\frac{1}{2}}v \quad (7)$$

the latter of which also leads to the scaling rules for the speed of light and photon’s frequency ($\nu = c/\lambda$):

$$c \rightarrow a^{-\frac{1}{2}}c \quad (8)$$

$$\nu \rightarrow a^{-\frac{3}{2}}\nu \quad (9)$$

in full consistency with the reasoning via \hbar . This offers another way to look at the scaling of frequency. If we are to interpret the photon as being redshifted in transit, the photon was emitted from a smaller-scale region that corresponded to a *faster light speed*. Therefore, not only the photon’s wavelength gets stretched out in transit, its speed gets gradually reduced on its journey to Earth. The latter effect which additionally suppresses the photon frequency has been missing from the conventional “redshift” explanation which demands simply that $\nu \rightarrow a^{-1}\nu$ (since $\lambda \rightarrow a\lambda$).

That the two considerations via \hbar and c respectively are in full consistency should not be a surprise. After all, the scaling rules are derived to make the Schrodinger equation for the hydrogen atom – which is under influence of Coulomb force – consistent. In other words, the scaling rules have been deliberately designed to make quantum mechanics *and* electromagnetism compatible.

The expanding universe is an excellent laboratory to study the effect of scale change on physical laws and physical constants. Whereas experimenters cannot adjust the scale of their apparatus at will (for example, Alice the physicist cannot “stretch” the size of the atoms in her ruler at her disposal), the universe offers such a scale change naturally. As we gaze progressively deeper into the night sky, we learn how our physical laws manifest

at progressively smaller scale – a scale which we cannot achieve at will on our laboratories. This is the view which we believe has been underemphasized in literature and which we advocate throughout this report.

C. The principle of scale invariance

We are now in a position to make some formal statement regarding the principle of scale invariance. We postulate that: *The equation describing a given law of physics has the same form in all choices of reference scale.*

Equivalently, *no test of the laws of physics can in any way distinguish between reference scales.* The latter statement means that a given physical property must transform universally for all systems under a scale transformation. Otherwise, one would be able to detect the effect of scale change by comparing the physical quantity for different systems.

Also note that the reference scale can be *local*; namely, the scale can be nonuniform across spacetime. In cosmology, the universe at large scale is observed to be isotropic and homogeneous (i.e., the cosmological principle), in which case the scale factor is global in space.

Upon a scale transformation, the fundamental constants adjust to cancel (or, compensate) the effect of the scale change on physical equations. “Intrinsic” properties of matter such as e , m_e , G are assumed to be unaffected, whereas “extrinsic” constants c and \hbar which describe motion in space get scaled. While the concept of a variable \hbar is easier to adopt, that of a variable c often incites great discomfort as it seems to contradict special relativity. There is, however, a fundamental distinction between constancy and universality. The value of c measured at a given point is a constant, independent of the reference frame, but this value needs not be universal for different points. It is the universality in c that we abandon while maintaining the constancy for c . We shall present an in-depth exposition on the compatibility of nonuniversal light speed with Lorentz invariance in Section IV. For the rest of this section, we shall mostly focus on the technical side of the scaling rules.

In 3-dimensional space, the scaling rules for physical quantities and constants are:

$$\vec{x} \rightarrow a\vec{x} \quad (10)$$

$$t \rightarrow a^{\frac{3}{2}}t \quad (11)$$

$$\vec{p} \rightarrow a^{-\frac{1}{2}}\vec{p} \quad (12)$$

$$E \rightarrow a^{-1}E \quad (13)$$

$$\hbar \rightarrow a^{\frac{1}{2}}\hbar \quad (14)$$

$$c \rightarrow a^{-\frac{1}{2}}c \quad (15)$$

$$m \rightarrow m \quad (16)$$

$$e \rightarrow e \quad (17)$$

$$\alpha \rightarrow \alpha \quad (18)$$

$$G \rightarrow G \quad (19)$$

where \vec{x} , t , \vec{p} , E , \hbar , c , m , e , α , G are coordinate, time, momentum, energy, Planck constant, speed of light, mass, electric charge, fine coupling constant, and gravitational constant respectively. In addition, the electromagnetic potential $A^\mu = (\varphi, \vec{A})$ and field $\vec{E} = -\vec{\nabla}\varphi - \frac{1}{c}\partial_t\vec{A}$, $\vec{B} = \vec{\nabla} \times \vec{A}$ and fermion field ψ scale as:

$$\varphi \rightarrow a^{-1}\varphi \quad (20)$$

$$\vec{A} \rightarrow a^{-1}\vec{A} \quad (21)$$

$$\vec{E} \rightarrow a^{-2}\vec{E} \quad (22)$$

$$\vec{B} \rightarrow a^{-2}\vec{B} \quad (23)$$

$$\psi \rightarrow a^{-\frac{3}{2}}\psi \quad (24)$$

It is straightforward to verify that the above scaling rules (10-24) leave the following equations and/or quantities unchanged:

- The partition function of QED (with $x^\mu = (ct, \vec{x})$)

$$\begin{aligned} Z &= \int \mathcal{D}\psi^\dagger \mathcal{D}\psi \mathcal{D}A^\mu \exp i \frac{S}{\hbar} \\ S &= \int dt d\vec{x} \mathcal{L} \\ \mathcal{L} &= \psi^\dagger \left(i\hbar \frac{\partial}{\partial t} + ic\vec{\alpha} \cdot \hbar \vec{\nabla} - mc^2 \beta \right) \psi \\ &\quad + e\psi^\dagger \left(-\varphi + \vec{\alpha} \cdot \vec{A} \right) \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu \end{aligned} \quad (25)$$

is unchanged since $\mathcal{L} \propto a^{-4}$ self-evidently. As the result, the Maxwell equations and the Schrödinger equation are also unchanged.

- The Einstein field equation

$$\mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

is unchanged since the curvature scales as a^{-2} , whereas $c^{-4} \propto a^2$, energy density $\rho c^2 \propto a^{-3}a^{-1} = a^{-4}$, and pressure $p = \text{force/area} = \text{mass.length.time}^{-2}/\text{area} \propto a^0 a^1 a^{-3}/a^2 = a^{-4}$.

Note that these are not results of a trivial exercise in dimensional consideration. Besides \hbar is measured in $kg.m^2.sec^{-1}$, there is nothing to further specify its scaling rule, which is also dependent on the number of spatial dimensions (see Appendix A).

The scaling rules also transform all forms of energy in the same fashion, $E \propto a^{-1}$. For example:

- The electromagnetic field energy $\frac{1}{2} \int d\vec{x} (\vec{E}^2 + \vec{B}^2) \propto a^3 (a^{-2})^2 = a^{-1}$ since $\vec{E} \propto a^{-2}$ and $\vec{B} \propto a^{-2}$.
- Photon energy $E = \hbar\nu \propto a^{\frac{1}{2}} a^{-\frac{3}{2}} = a^{-1}$.
- Energy of a massive object: $E = mc^2 \propto a^{-1}$.

- Energy levels of the hydrogen atom $E_{jn} = -\frac{m_e e^4}{2\hbar^2 n^2} \left[1 + \frac{\alpha^2}{n^2} \left(\frac{n}{j+\frac{1}{2}} - \frac{3}{4} \right) \right] \propto a^{-1}$.

The requirement that all forms of energy obey a universal scaling rule is important. Imagine that Alice the physicist lets the photon emitted from the hydrogen atom bounce off a pair of mirrors on the opposite sides of her lab for a long period – during which time the space surrounding her lab has enlarged – then brings back the photon in contact with the atom. Since the “redshifted” photon’s energy has decreased, if the atom’s energy levels remain unchanged, the atom will not be able to re-absorb the photon and she would be able to tell the space expansion using the apparatus *within* her lab, in violation of the scale invariance principle. Only if its energy levels decrease by the same scale factor does the atom happily re-absorb the photon, denying Alice a mean to tell whether the space has expanded.

Let us stress that our consideration is unconventional. The standard cosmological paradigm discriminates radiation from matter. “Doppler theft” only affects photons but leaves nonrelativistic matter untouched. *This practice is questionable.* According to scale invariance requirement, all forms of energy should obey a universal scaling rule, $E \propto a^{-1}$.

The fact that our gravitationally bounded Milky Way is insulated from cosmic space expansion has important consequences as will be discussed in Section V G. *But whether or not space undergoes an actual expansion is irrelevant in our logics. What matters is the **restoration of scale invariance** - a principle that we adhere to - to the established laws of physics.*

It should be clear by now that the scaling rules offer an “heuristic” resolution to the scaling mismatch in Friedmann’s equation – the issue that we raised in the Motivation. By letting $c \propto a^{-\frac{1}{2}}$ and $t \propto a^{\frac{3}{2}}$, all three terms in Friedman’s equation, $(\dot{a}/a)^2 = 8\pi G\rho/3 - kc^2/a^2$, now obey the same scaling; they all are proportional to a^{-3} . Yet this *ad hoc* treatment is flawed since Friedman’s equation was obtained from the assumption of a constant c . *Friedmann’s equations need be abandoned* and we shall derive the resolution *rigorously* and consistently from Einstein’s field equation in Section III.

D. The generalized laws of physics in the evolutionary universe

We propose the following correspondence procedure: the fundamental laws of physics are generalized in such a way that in the absence of scale variation the laws recover their established form. To do so, the speed of light and Planck constant in physical equations are replaced by their scale-dependent counterparts, namely:

$$c \rightarrow a^{-\frac{1}{2}}(x)c_0 \quad (26)$$

$$\hbar \rightarrow a^{\frac{1}{2}}(x)\hbar_0 \quad (27)$$

where $a(x)$ is the local scale factor at spacetime point x , c_0 and \hbar_0 the values of speed of light and Planck constant at scale $a = 1$. It is crucial that c and \hbar be dependent *solely on scale a* , not t and/or \vec{x} .

For example, the generalized Einstein field equation is:

$$\mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R} = \frac{8\pi G}{c_0^4}a^2(x)T_{\mu\nu} \quad (28)$$

Similarly, the energy and momentum operators in quantum mechanics are:

$$\hat{E} \rightarrow i\hbar_0 a^{\frac{1}{2}}(x)\frac{\partial}{\partial t} \quad (29)$$

$$\hat{\vec{p}} \rightarrow -i\hbar_0 a^{\frac{1}{2}}(x)\vec{\nabla} \quad (30)$$

The validity of our correspondence procedure is by no mean guaranteed. The generalized laws must stand falsification.

Friedmann’s equations and the conservation of energy are “derived” laws. They cannot be generalized via the correspondence procedure. Rather, they will need be rederived from the fundamental laws. The revision of Friedmann’s equations is the focus of section III. The issue with (non)conservation of energy will be discussed in Section VII A.

III. THE SCALE-INVARIANT MODEL OF COSMOLOGY

Our strategy is comprised of three steps: (1) Specify a metric that respects scale invariance and Lorentz invariance for an isotropic and homogeneous universe; (2) Apply the generalized Einstein field equation to the metric to obtain the evolution equations for the metric over the cosmic time; (3) Find the solutions to the evolution equations.

A. The modified Robertson-Walker metric

The Robertson-Walker (RW hereafter) metric for a homogenous and isotropic universe has been determined to be

$$ds^2 = c^2 dt^2 - a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right] \quad (31)$$

$$d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2 \quad (32)$$

where a is the global scale factor of the universe and is a function of the cosmic time t only, k the curvature determining the shape of the universe. Recall that the RW metric assumes a constant speed of light.

Exploiting the correspondence procedure (26), we arrive at the *modified Robertson-Walker metric*:

$$ds^2 = \frac{c_0^2}{a(t)} dt^2 - a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right] \quad (33)$$

in which c_0 is the speed of light measured at our current time. The modified RW metric is scale invariant and locally Lorentz invariant. We stress that the value of c_0 must not be identical with the value measured using light from Earth-bound sources, i.e., $300,000 \text{ km} \cdot \text{sec}^{-1}$. This is because our Solar system is gravitationally bounded whereas the modified RW metric describes the gravitationally unbounded regions, e.g., beyond the outskirts of the Milky Way. The value of c_0 is not known at the moment, but in future applications we shall measure distances in term of light years to avoid the problem.

B. The modified Friedmann equations

The application of the Einstein field equation to the modified RW metric is straightforward. The detailed calculation is presented in Appendix B for the keen-eyed reader. We only note in passing that some extra care is needed for the term c^4 and the stress-energy tensor in the RHS of the Einstein equation since they are explicitly scale dependent.

Here, we only quote the results – Eqs(B35) and (B36) of Appendix B – which we shall call the *modified Friedmann equations*:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho - \frac{kc_0^2}{a^3} \quad (34)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(2\rho + 3\frac{p}{c_0^2}a\right) + \frac{kc_0^2}{2a^3} \quad (35)$$

Several interesting points are in order. On the conceptual side:

- Scale invariance is graciously restored! Both of the newly obtained equations automatically satisfy the scaling rules: $\rho \propto a^{-3}$, $p \propto a^{-4}$. The problem of scaling mismatch raised in our Motivation section is resolved.
- *Superficially*, the traditional Friedmann equations: $\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^3}$ and $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho$ (in the “dust” case) appear invariant under the scale change $\rho \propto a^{-3}$ and additionally that $c \propto a^{-\frac{1}{2}}$. Nonetheless this “rescue” is fatally flawed because Friedmann’s equations assumed c to be scale independent! The only logically consistent treatment to the scaling mismatch is to abandon the traditional Friedmann equations and rederive them from Einstein’s field equation with *ab initio* scale dependency for c .

On the technical side:

- The first equation only slightly differs from the first Friedmann equation in their final term. Yet this difference plays a crucial role to resolve a series of difficulties in the Big Bang model. We shall present its resolutions in Section VI. An aside note: the

first equation also looks as if it could have been obtained from the traditional Friedman equation by replacing c by $c_0 a^{-\frac{1}{2}}$. This coincidence is deceptive, however; the scale dependency for c must be recognized *ab initio* in the modified RW metric and Einstein’s field equation.

- The second equation, in contrast, looks drastically different from Friedmann’s acceleration equation. Two noteworthy features in Eq(35): (a) An additional factor of 2 in front of ρ ; (b) An appearance of the curvature k in the RHS. Indeed, the second equation has no Newtonian counterpart. We shall shortly see (Eq(42) below) that the modified acceleration equation adopts a simpler form.
- Even the flat universe, $\kappa = 0$, is not the Einstein-de Sitter one. The speed of light varies with scale in our consideration.

C. Solution to the modified Friedmann equations

The modified Friedmann equations (34, 35) admit the following solution:

$$a = \left(\frac{t}{t_0}\right)^{\frac{2}{3}} \quad (36)$$

$$\rho = \rho_0 a^{-3} = \rho_0 t_0^2 t^{-2} \quad (37)$$

$$p = p_0 a^{-4} = p_0 t_0^{\frac{8}{3}} t^{-\frac{8}{3}} \quad (38)$$

a fact easy to verify. For the first modified Friedmann equation, Eq(34):

$$\begin{aligned} \frac{4}{9}t^{-2} &= \frac{8\pi G}{3}\rho_0 t_0^2 t^{-2} - kc_0^2 t_0^2 t^{-2} \\ t_0 &= \frac{2}{3} \left(\frac{8\pi G}{3}\rho_0 - kc_0^2 \right)^{-\frac{1}{2}} \end{aligned} \quad (39)$$

For the second modified Friedmann equation, Eq(35):

$$\begin{aligned} -\frac{2}{9}t^{-2} &= -\frac{4\pi G}{3} \left(2\rho_0 t_0^2 t^{-2} + 3\frac{p_0}{c_0^2} t_0^2 t^{-2} \right) + \frac{1}{2}kc_0^2 t_0^2 t^{-2} \\ t_0 &= \frac{\sqrt{2}}{3} \left[\frac{4\pi G}{3} \left(2\rho_0 + 3\frac{p_0}{c_0^2} \right) - \frac{1}{2}kc_0^2 \right]^{-\frac{1}{2}} \end{aligned} \quad (40)$$

There exists a precise connection enforced by Eqs(39) and (40). Equating t_0 of the two equations and arranging terms, we get:

$$\rho_0 + 3\frac{p_0}{c_0^2} = 0$$

or, equivalently (recall that $c = c_0 a^{-\frac{1}{2}}$, $\rho = \rho_0 a^{-3}$, $p = p_0 a^{-4}$):

$$p = -\frac{1}{3}\rho c^2 \quad (41)$$

We deduce that:

- There is a strict equality between pressure and density dictated by Eq(41).
- Interestingly, *pressure is negative*. Yet it is not sufficient to suppress the density term in Eq(35) and turn the acceleration positive.
- Using (41) and (34), the second modified Friedman equation, Eq(35), can be simplified as:

$$\begin{aligned}\frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}\rho + \frac{kc_0^2}{2a^3} \\ &= -\frac{1}{2}\frac{\dot{a}^2}{a^2}\end{aligned}\quad (42)$$

which enforces that $\ddot{a} < 0$.

- There is an upper bound for the curvature, as deduced from (42):

$$k \leq \frac{8\pi G}{3} \frac{\rho_0}{c_0^2} \quad (43)$$

- The deceleration parameter

$$q(t) \equiv -\frac{\ddot{a}a}{\dot{a}^2} = \frac{1}{2} \quad (44)$$

at all time, due to Eq(42). This means that the universe expansion has always been *decelerating*.

We must note in advance that the studies of Type Ia supernovae since 1998 announced to have found an “acceleration” in the universe expansion [12]. *These conclusions are questionable, however.* There is a problem in determining the redshift values for the supernovae in those studies. Specifically, the traditional redshift formula is no longer applicable; because of the scale-dependent speed of light, a modified formula is needed. This topics will be investigated in full details in Section V.

To summarize, we obtained the modified Friedmann equations and the pressure-density connection:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho - \frac{kc_0^2}{a^3} \quad (45)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho + \frac{kc_0^2}{2a^3} \quad (46)$$

$$p = -\frac{1}{3}\rho c^2 \quad (47)$$

$$c = c_0 a^{-\frac{1}{2}}$$

Equations (45, 46) are of the preferred form thanks to their simplicity. If we also think of the nonzero spatial curvature as a form of “density”

$$\rho_{curv} = -\frac{3kc_0^2}{8\pi G a^3} \quad (48)$$

which scales similarly to ρ as a^{-3} , the modified Friedmann equations neatly become:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho_{total} \quad (49)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho_{total} \quad (50)$$

$$\rho_{total} = \rho + \rho_{curv}$$

These equations are analogous to their Newtonian counterpart without pressure and curvature.

D. Equation of state

Taking derivative with respect to t on Eq(34) we get:

$$2\frac{\dot{a}\ddot{a}}{a^2} - 2\frac{\dot{a}^3}{a^3} = \frac{8\pi G}{3}\dot{\rho} - 3kc^2\frac{\dot{a}}{a^4}$$

which, combined with Eq(42) yields:

$$\begin{aligned}\frac{8\pi G}{3}\dot{\rho} &= \frac{\dot{a}}{a} \left(2\frac{\ddot{a}}{a} - 2\frac{\dot{a}^2}{a^2} - 3\frac{kc^2}{a^3} \right) \\ &= \frac{\dot{a}}{a} \left(-\frac{8\pi G}{3}\rho + \frac{kc_0^2}{a^3} - \frac{16\pi G}{3}\rho + 2\frac{kc^2}{a^3} - 3\frac{kc^2}{a^3} \right) \\ &= -\frac{24\pi G}{3}\frac{\dot{a}}{a}\rho\end{aligned}$$

or

$$\dot{\rho} = -3\frac{\dot{a}}{a}\rho \quad (51)$$

This is the equation of state which automatically satisfies the scaling rule for density as anticipated:

$$\rho \propto a^{-3} \quad (52)$$

The equation of state is unique and does not involve the pressure term, which in turn satisfies Eq(41).

E. The age formula

The solution (36) directly leads to

$$\frac{\dot{a}}{a} = \frac{2}{3t}$$

or (recall that $H \equiv \dot{a}/a$)

$$t = \frac{2}{3H}$$

This result is also derivable *solely* from the scaling property of time. From $t \propto a^{\frac{3}{2}}$ we deduce that

$$\frac{\dot{a}}{a} = \left(\frac{\dot{a}}{a} \right)_{t=t_0} a^{-\frac{3}{2}} = H_0 a^{-\frac{3}{2}}$$

or

$$t_0 = \frac{1}{H_0} \int_0^1 da \sqrt{a} = \frac{2}{3H_0} \quad (53)$$

Thus our model, regardless of the curvature κ , yields a *universal* formula for the age of the universe. Although this age formula is identical to that in Einstein-de Sitter universe, our result holds for all shapes of the universe. Indeed, even for $\kappa = 0$ our universe is not Einstein-de Sitter since the speed of light varies with respect to scale in our model, whereas in Einstein-de Sitter universe c was a constant.

For the sake of later comparison, let us quote the age formula for the Λ CDM model in a flat space:

$$\frac{\dot{a}}{a} = H_0 \left(\Omega_M \frac{1}{a^3} + \Omega_\Lambda \right)^{\frac{1}{2}} \\ t_0 = \int_0^1 \frac{da}{H_0 a \sqrt{\Omega_M \frac{1}{a^3} + \Omega_\Lambda}} = \frac{2}{3H_0} \frac{\tanh^{-1} \sqrt{\Omega_\Lambda}}{\sqrt{\Omega_\Lambda}} \quad (54)$$

which involves three parameters H_0 , Ω_M , Ω_Λ , beside $\Omega_{curv} = 1 - \Omega_M - \Omega_\Lambda = 0$, in contrast to our age formula (53) which requires only one parameter H_0 .

IV. COMPATIBILITY OF NONUNIVERSAL SPEED OF LIGHT WITH LORENTZ INVARIANCE

The possibility of variation in the speed of light is a touchy issue for every consideration that attempts a variable light speed. Whereas a variable Planck constant would be easier to accept, a variable speed of light appears to frontally assault upon one vital pillar of modern physics, the special theory of relativity. In particular, it stokes a fear that whether causality would be violated if the speed of light is allowed to vary.

The speed of light is not universal in our approach; it can vary from one point to another (indeed, c *must* vary if the scale factor is inhomogeneous on the spacetime manifold). However, two important points stand out:

1. The speed of light at a given point and a given instant is independent of the reference frame one resides in to measure c at *that* given point and *that* given instant.
2. The speed of light at a given point and a given instant is the *maximum velocity* that any physical object can attain when it travels across *that* given point and *that* given instant.

Point (1) is the very content of Einstein's postulate of constant light speed. This postulate is fully retained in our approach. Point (2) is a direct outcome of Einstein's postulate of constant light speed and his relativity principle. This result is also fully preserved in our approach. The only new element in these statements is the *locality*

in the value of c . There is an important distinction in the nomenclature: constancy versus universality. Whereas the value of c is constant, it needs not be universal.

We attribute the uneasiness in accepting the notion of nonuniversal speed of light to a prejudice which is rooted in the false belief in a *global* validity of special relativity. It is important to note that special relativity has lost its status of a global symmetry for the 4-dimensional manifold with the birth of general relativity. General relativity has relegated Lorentz invariance to hold only locally: in a curved spacetime, at a given point, one can find a family of reference frames (that is, the free-fall frames) to locally remove the effect of gravity (that is, the curvature of spacetime). Lorentz invariance holds, among those frames, within the vicinity of that point. The "localization" of Lorentz invariance directly leads us to the preceding statements (1) and (2). In curved spacetime, each point acquires its own metric characterized by its *own* value of c . The variation of c manifests when we compare its values measured in distinct spacetime regions of separate scales.

In support of our logics, the modified Friedmann equations derived in Appendix B provides an excellent case study. Our modified RW metric coupled with the Einstein field equation *effortlessly* produces the modified Friedmann equations. Indeed, for either metric – the traditional RW metric or the modified RW one – the geometric Einstein tensor in the comoving coordinates $(x^0, x^1, x^2, x^3) = (\eta, \chi, \theta, \phi)$ adopts an identical form! For the reader's convenience, we quote the result here, Eqs(B12, B13):

$$G_0^0 = \frac{3}{R^2} \left(\frac{R'^2}{R^2} + \kappa \right) \\ G_k^k = \frac{1}{R^2} \left(2 \frac{R''}{R} - \frac{R'^2}{R^2} + \kappa \right)$$

which are common for both metrics. That is to say, the geometrical structure in Einstein's field equation *admits* the nonuniversal speed of light. Any objection to the nonuniversality of c would need to pinpoint the error in the modified RW metric and/or the modified Friedmann equations.

Causality means that a pair of events that lie outside the lightcone of each other cannot influent each other in any physical way. In relativity theories, causality is absolute: no physical transformation of coordinates can alter the causal connection (or the lack thereof) in a pair of events. The set of null geodesics (along which light travel) that pass through one event separate the space-like region from the timelike region with respect to that event. Under a continuous local scale change, the spacetime manifold gets deformed such that the spacelike region and the timelike region do not mix. Causality thus remains absolute upon a local scale transformation.

In our approach the variation in c (and \hbar) arises via its *endogenous* dependency on the spatial scale factor. This is a radical departure from variable-speed-of-light

attempts that introduced a dependency of c explicitly on space/time coordinates. For example, in some of those attempts, c was allowed to acquire a dynamics governed by some quantum mechanical field. In our approach, the variation in c and \hbar is a kinematic effect, instead of a dynamical one. These constants have no dynamics. There is no room for terms such as \dot{c} or $\vec{\nabla}\hbar$ or the like which could otherwise inflict structural damages to existing physical laws. As the result, the fundamental laws are neatly preserved and are readily extendible – via the correspondence procedure – to describe physics in the expanding universe.

We adopt the relativist’s view: The scale invariance principle enforces the change in c (and \hbar). There is no need to devise a mechanism to account for the variation in these constants. An apt analogy is warranted: prior to the advance of special relativity, Lorentz-FitzGerald’s length contraction and the Lorentz transformation had been devised to account for Michelson-Morley’s experiment. Lorentz attributed these effects to some yet-to-discover mechanism, such as the interaction between electrons in the ruler and clock with an ether. In contrast, Einstein considered these effects a matter of principle – the principle of relativity and the postulate of constant light speed – both of which together set a strict form for the Lorentz transformation (and the length contraction). Indeed Einstein insisted on using the term Relativitätsprinzip (i.e., a *descriptive* approach) before finally adopting Relativitätstheorie (i.e., an *explanatory* approach), a term coined by Planck (e.g., see [20]).

We follow Einstein’s pioneer footsteps in this regard: the variation of c and \hbar is a matter of principle – the principle of scale invariance. It is not necessary to design a mechanistic model to explain the effect of c and \hbar variations. Yet the variations in c and \hbar are not arbitrary; they are set by strict rules guided by the scale invariance requirement: $c \propto a^{-\frac{1}{2}}$, $\hbar \propto a^{\frac{1}{2}}$.

Relativity principle imposes a *powerful constraint* on the admissible structure of physical laws – the laws must be in a covariant form. Scale invariance should be viewed with the same spirit: it enforces a *powerful requirement* about the admissible form of physical laws – the constants of Nature must adapt to the local scale.

In [8], Ellis posed a checklist of conceptual issues to be satisfactorily handled by physical theories that deal with variable light speed. It is important that we assess the soundness of our approach against his legitimate criteria. The most noteworthy points are:

- The speed of light and measurement: It was suggested in [8] that since speed of light is not dimensionless, a variation in c is not meaningful as “one can change the apparent speed of light to any desired value by changing... the units used”. We disagree with this assertion. In the context of cosmology, if we are to play the role of Bob the spectator, i.e., to compare the values of c different places, we must ensure that *one* set of units be used. That is to say, the *relative* variation in c is a meaningful

concept and a detectable effect.

- The speed of light and the Lorentz group: Our approach preserves Lorentz invariance, albeit locally.
- The speed of light and Maxwell’s equations: Maxwell’s equations, like other fundamental laws of physics, retain their form in our approach. The only change is the replacement of c by $c_0 a^{-\frac{1}{2}}$ and \hbar by $\hbar_0 a^{\frac{1}{2}}$ per correspondence rules (26, 27).
- The speed of light and the integrated wholeness: [8] correctly requests that “when a quantity is changed from a constant to a function, obtaining the correct dynamic equations requires doing the variation with that quantity treated as a variable *ab initio*”. In our cosmological model, the variable c is introduced *ab initio* in the modified RW metric and the modified Friedmann equations are derived directly from Einstein’s field equations with *ab initio* recognition of the variable c .
- The speed of light and dynamical equations: This is the most interesting point. As we elaborated previously, in our approach the variation in c and \hbar is a kinematic effect instead of a dynamical one. That is to say, the dynamics of c and \hbar is *absent*. There is no \dot{c} or $\vec{\nabla}\hbar$ or the like. There does not exist the need to devise a mechanical (or field theoretical) model to explain the variations in these constants. Let us emphasize again that a task to find a mechanism that accounts for variations in c and \hbar is analogous to the task that Lorentz undertook to explain the length contraction and time dilation (although Lorentz had not been aware of the latter effect prior to 1905) via some yet-to-discover interaction of the ether with matter, an interaction that would shrink the moving ruler and slow the moving clock. With the advance of special relativity, we come to appreciate that a dynamics for length contraction and time dilation is unnecessary. Likewise, by introducing the variations of c and \hbar via scale invariance principle, we *bypass the whole enterprise of such a mechanistic design*.

V. IMPLICATIONS IN COSMOGRAPHY

In this Section, we shall derive a series of revisions arising from scale-dependent c to several formulae used in cosmography. For the sake of comparison, the derivations are presented both in the traditional framework and in our approach. Correspondingly, the calculations are based on the traditional RW metric and the modified RW metric. Of these results, the modified Lemaitre redshift formula is the central one that has ramifications in all the rest.

A. Modification to Lemaitre's redshift formula

1. For the traditional RW metric:

Light wave travels in the null geodesics, $ds^2 = 0$. In the traditional RW metric:

$$ds^2 = c^2 dt^2 - a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right] \quad (55)$$

the null geodesics for a light wave traveling from a galaxy toward Earth (thus $d\Omega = 0$) is:

$$\frac{c dt}{a(t)} = \frac{dr}{\sqrt{1 - kr^2}} \quad (56)$$

Denote t_e and t_o the emission and observation timepoints of the light wave, and r_e the comoving distance of the galaxy from Earth. We have:

$$\int_{t_e}^{t_o} \frac{c dt}{a(t)} = \int_{r_e}^0 \frac{dr}{\sqrt{1 - kr^2}} \quad (57)$$

The next wavecrest to leave the galaxy at $t_e + \delta t_e$ and arrives at Earth at $t_o + \delta t_o$ satisfies:

$$\int_{t_e + \delta t_e}^{t_o + \delta t_o} \frac{c dt}{a(t)} = \int_{r_e}^0 \frac{dr}{\sqrt{1 - kr^2}} \quad (58)$$

Subtracting the two equations yields:

$$\frac{c \delta t_o}{a(t_o)} = \frac{c \delta t_e}{a(t_e)} \quad (59)$$

The observed wavelength is related to the emitted wavelength by (since $\lambda_{e/o} = c/\nu \propto c \delta t_{e/o}$):

$$\frac{\lambda_o}{\lambda_e} = \frac{c \delta t_o}{c \delta t_e} = \frac{a(t_o)}{a(t_e)} \quad (60)$$

The *wavelength-based redshift parameter* is

$$z \equiv \frac{\lambda_o - \lambda_e}{\lambda_e} \quad (61)$$

We thus have

$$z = \frac{a(t_o)}{a(t_e)} - 1 \quad (62)$$

or, setting $a(t_o) = 1$, the traditional Lemaitre redshift formula is:

$$1 + z = a^{-1}(t_e) \quad (63)$$

2. For the modified RW metric:

Recall that the modified RW metric, with c_0 the speed of light at our current time, is:

$$ds^2 = c_0^2 \frac{dt^2}{a(t)} - a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right] \quad (64)$$

The null geodesics for a light wave traveling radially is:

$$\frac{c_0 dt}{a^{\frac{3}{2}}(t)} = \frac{dr}{\sqrt{1 - kr^2}} \quad (65)$$

leading to:

$$\int_{t_e}^{t_o} \frac{c_0 dt}{a^{\frac{3}{2}}(t)} = \int_{r_e}^0 \frac{dr}{\sqrt{1 - kr^2}} \quad (66)$$

$$\int_{t_e + \delta t_e}^{t_o + \delta t_o} \frac{c_0 dt}{a^{\frac{3}{2}}(t)} = \int_{r_e}^0 \frac{dr}{\sqrt{1 - kr^2}} \quad (67)$$

Subtracting the two equations yields:

$$\frac{\delta t_o}{a^{\frac{3}{2}}(t_o)} = \frac{\delta t_e}{a^{\frac{3}{2}}(t_e)} \quad (68)$$

The observed *frequency* is related to the emitted *frequency* by

$$\frac{\nu_o}{\nu_e} = \frac{\delta t_e}{\delta t_o} = \frac{a^{\frac{3}{2}}(t_e)}{a^{\frac{3}{2}}(t_o)} \quad (69)$$

The *frequency-based redshift parameter* is

$$z \equiv \frac{\nu_e - \nu_o}{\nu_o} \quad (70)$$

leading to the *modified Lemaitre redshift formula*:

$$1 + z = a^{-\frac{3}{2}}(t_e) \quad (71)$$

We stress that in observational astronomy *it is the shift in frequency - not wavelength - in the light spectrum that is used to determine z* . In the case of constant speed of light, the ν -based and λ -based redshift formulae are identical. This is no longer the case if c varies. The distinction between the two formulae is crucial in our model given that $c(t_o) \neq c(t_e)$ if $a(t_o) \neq a(t_e)$ as often is the case. Once again, the correct definition for z is Eq(70), and the modified Lemaitre redshift formula (71) applies. The modified Lemaitre redshift formula plays a key role in other subsequent modifications in the rest of this Section.

B. The modified Hubble law and the corrected value for Hubble constant

Denote $d = c_0 \cdot (t_o - t_e)$ as the distance from Earth to a galaxy. For small z and d , the Taylor expansion for the modified Lemaitre redshift formula (71)

$$\begin{aligned} 1 + z &= a^{-\frac{3}{2}}(t_e) \\ &= (1 - H_0(t_o - t_e) + \dots)^{-\frac{3}{2}} \\ &= 1 + \frac{3}{2}H_0(t_o - t_e) + \dots \\ &= 1 + \frac{3}{2}H_0 \frac{d}{c_0} \end{aligned}$$

produces the *modified Hubble law*:

$$z = \frac{3}{2} H_0 \frac{d}{c_0} \quad (72)$$

Compared with the conventional Hubble law, where the speed of light is explicitly restored:

$$z = H_0 \frac{d}{c} \quad (73)$$

the modified Hubble law *acquires an additional factor of 3/2*. A very important consequence is that the Hubble constant – determined from low- z objects – has been *inadvertently overestimated by a factor of 3/2*. That is to say, in the plot of z versus d/c , if the slope is 71, the actual Hubble number H_0 should – per Eq(72) – be only $\frac{2}{3} \times 71 \approx 47$ rather than 71 as the traditional Hubble law (73) would dictate.

Remarkably this correction reduces the Hubble value and – from our universal age formula $t_0 = 2/3H_0^{-1}$ – exactly amounts to bringing the age estimate back up to 13.8 Gyr !

A too high Hubble value – at 71 – was one of the motivations for cosmologists to have sought an alternative to the Einstein-de Sitter model (which would have predicted too young a universe, 9.2 Gyr , for the oldest stars to have formed). The widely adopted alternative is the negative cosmological constant which accommodates a deceleration phase followed by an acceleration phase such that $t_0 \approx H_0^{-1} \approx 13.8\text{ Gyr}$ for $H_0 = 71$. In the light of the 3/2-reduction in H_0 value, this deceleration-acceleration account is no longer advantageous.

Another important consequence is that the reduced Hubble value also leads to a lower critical density (since $\rho_c \propto H_0^2$) than previous thought by a factor of $(3/2)^2$.

The reduction of 3/2, however, is not the end of the story yet. We shall momentarily show that there are two additional sources of correction: (i) The first is a new distance-redshift formula in our model as compared to that in the ΛCDM model; (ii) The other is an extra modification factor when one converts the luminosity distance into the proper distance, a practice of importance for high- z objects, such as Type Ia supernovae.

C. The modified distance-redshift relationship

1. For ΛCDM model:

Let us first rederive the distance-redshift relationship within ΛCDM model for a flat universe. The traditional Friedmann equation is being recast as:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3} (\rho_M + \rho_\Lambda) = H_0^2 \left[\Omega_M \frac{1}{a^3} + \Omega_\Lambda \right] \quad (74)$$

in which

$$\Omega_M \equiv \frac{\rho_M(t_0)}{\rho_c(t_0)} = \frac{8\pi G}{3H_0^2} \rho_M(t_0) \quad (75)$$

$$\Omega_\Lambda \equiv \frac{\rho_\Lambda(t_0)}{\rho_c(t_0)} = \frac{8\pi G}{3H_0^2} \rho_\Lambda \quad (76)$$

$$\Omega_M + \Omega_\Lambda = 1 \quad (77)$$

For the almost flat space, the proper distance from the galaxy and Earth is:

$$r \approx \chi = \int_{t_e}^{t_o} \frac{c dt}{a(t)} \quad (78)$$

Applying the traditional Lemaitre redshift formula

$$1 + z = a^{-1}(t) \Rightarrow dz = -\frac{\dot{a}}{a^2} dt \quad (79)$$

we obtain:

$$r = \int_0^z \frac{c dz'}{(\dot{a}/a)(z')} \quad (80)$$

or

$$\frac{r}{c} = \frac{1}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_M(1+z')^3 + \Omega_\Lambda}} \quad (81)$$

For low z , it recovers the Hubble law as expected. Two special cases of interest:

- The Einstein-de Sitter universe ($\Omega_M = 1, \Omega_\Lambda = 0$) corresponds to

$$\frac{r}{c} = \frac{2}{H_0} \left(1 - \frac{1}{\sqrt{1+z}} \right) \quad (82)$$

- The de Sitter universe ($\Omega_M = 0, \Omega_\Lambda = 1$) corresponds to

$$\frac{r}{c} = \frac{z}{H_0} \quad (83)$$

2. For our model:

Let us derive the distance-redshift relation in our model. We restore the curvature k and explicitly denote c_0 as current day's speed of light; the modified Friedmann equation reads:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3} \rho_M - \frac{kc_0^2}{a^3} = H_0^2 \left[\Omega_M - \frac{kc_0^2}{H_0^2} \right] \frac{1}{a^3} = H_0^2 \frac{1}{a^3} \quad (84)$$

in which

$$\Omega_M \equiv \frac{\rho_M(t_0)}{\rho_c(t_0)} = \frac{8\pi G}{3H_0^2} \rho_M(t_0) \quad (85)$$

$$\Omega_M - \frac{kc_0^2}{H_0^2} = 1 \quad (86)$$

The proper distance in the almost flat space is:

$$r \approx \chi = \int_{t_e}^{t_o} \frac{c_0 dt}{a^{\frac{3}{2}}(t)} \quad (87)$$

Applying the modified Lemaitre redshift formula

$$1 + z = a^{-\frac{3}{2}}(t) \Rightarrow dz = -\frac{3}{2} \frac{\dot{a}}{a^{\frac{5}{2}}} dt \quad (88)$$

we obtain the *modified distance-redshift relationship*:

$$r = \frac{2}{3} \int_0^z \frac{c_0 dz'}{(\dot{a}/a)(z')} = \frac{2c_0}{3H_0} \int_0^z \frac{dz'}{1+z'} \quad (89)$$

or

$$\frac{r}{c_0} = \frac{2}{3H_0} \ln(1+z) \quad (90)$$

Two important features to note:

- Only H_0 appears in the RHS of (90). The mass density and the spatial curvature are not involved in the formula. This helps make the fitting to observational data parsimonious and universal.
- The correction factor of $\frac{3}{2}$ for H_0 is again manifest. In the low- z limit, the modified distance-redshift formula indeed recovers the modified Hubble law as expected.

D. The modified luminosity-redshift relationship

So far we focused on the proper distance for cosmological objects. However, physical distance is not directly measurable but must be deduced from angular distance or luminosity distance. We shall work out the conversion from the proper distance to the luminosity distance in our model.

1. For Λ CDM model:

Consider a source located at the comoving coordinate χ_e with total luminosity L . The energy output at emission time t_e within the window δt_e is given by

$$\Delta E_e = L \delta t_e \quad (91)$$

As the photons traversed the distance, their energy get “Doppler thieved” by the scale factor $a(t_o)/a(t_e)$. Therefore, at the moment of observation, the observed energy will be

$$\Delta E_o = \Delta E_e a(t_e) \quad (92)$$

The physical area of the sphere centered at χ_e and radius $r(z)$ to be crossed by photons today is

$$S(z) = 4\pi r^2(z) \quad (93)$$

The visible brightness (energy flux at observer’s position) equals

$$J = \frac{\Delta E_o}{S(z)\Delta t_o} = \frac{\Delta E_e a(t_e)}{S(z)\Delta t_o} = \frac{(L\Delta t_e)a(t_e)}{S(z)\Delta t_o} = \frac{L}{S(z)} a^2(t_e) \quad (94)$$

in which we have made use of

$$\frac{\delta t_o}{a(t_o)} = \frac{\delta t_e}{a(t_e)} \quad (95)$$

Applying the traditional Lemaitre redshift formula $a(t_e) = (1+z)^{-1}$ we obtain

$$J = \frac{L}{4\pi r^2(z)} (1+z)^{-2} \quad (96)$$

The photometric distance d_L is defined via

$$J = \frac{L}{4\pi d_L^2(z)} \quad (97)$$

or

$$d_L = (1+z)r(z) \quad (98)$$

Coupled with

$$\frac{r}{c} = \frac{1}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_M(1+z')^3 + \Omega_\Lambda}} \quad (99)$$

we get

$$\frac{d_L}{c} = \frac{1}{H_0} (1+z) \int_0^z \frac{dz'}{\sqrt{\Omega_M(1+z')^3 + \Omega_\Lambda}} \quad (100)$$

For the sake of later comparison, the Einstein-de Sitter universe has

$$\frac{d_L}{c} = \frac{2}{H_0} (1+z) \left(1 - \frac{1}{\sqrt{1+z}} \right) \quad (101)$$

2. For our model:

In the “Doppler theft”, although the speed at which photons arrive at the observer gets changed, the Planck constant also acquires a variation in *exact cancellation* that of c (recall that $c \propto a^{-\frac{1}{2}}$ and $\hbar \propto a^{\frac{1}{2}}$ such that $E = \hbar c/\lambda \propto a^{-1}$). Therefore, the photon energy still gets reduced by the scale factor $a(t_o)/a(t_e)$.

The visible brightness (energy flux at observer’s position) equals

$$J = \frac{\Delta E_o}{S(z)\Delta t_o} = \frac{\Delta E_e a(t_e)}{S(z)\Delta t_o} = \frac{(L\Delta t_e)a(t_e)}{S(z)\Delta t_o} = \frac{L}{S(z)} a^{\frac{5}{2}}(t_e) \quad (102)$$

in which we have made use of

$$\frac{\delta t_o}{a^{\frac{3}{2}}(t_o)} = \frac{\delta t_e}{a^{\frac{3}{2}}(t_e)} \quad (103)$$

Applying the modified Lemaitre redshift formula $a(t_e) = (1+z)^{-\frac{2}{3}}$ we obtain

$$J = \frac{L}{4\pi r^2(z)} (1+z)^{-\frac{5}{3}} \quad (104)$$

The photometric distance now becomes

$$d_L = (1+z)^{\frac{5}{6}} r(z) \quad (105)$$

Coupled with

$$\frac{r}{c_0} = \frac{2}{3H_0} \ln(1+z) \quad (106)$$

we get the *modified photometric distance-redshift relationship*:

$$\frac{d_L}{c_0} = \frac{2}{3H_0} (1+z)^{\frac{5}{6}} \ln(1+z) \quad (107)$$

This is the central formula of our model to assess Type Ia supernovae data. It is universally applicable to all shapes of the universe (flat/open/closed).

E. A critical analysis of Type Ia supernovae

In 1998, it was announced that Type Ia supernova of high-redshift reveal a surprising behavior. The supernovae appear unexpectedly faint – and thus farther – for cosmological objects of the same redshift [12]. This section and the next provide a critical look at the Type Ia supernova data in the light of our model.

Type Ia supernovae are objects with stable luminosity, hence the name standard candles. Astronomers exploit this property to deduce the luminosity distance d_L from Earth of these objects. Their redshift values z , as usual, are independently obtained from the Doppler effect. For Type Ia supernovae, the curve d_L against z bends upward toward the high- z end. This striking behavior warranted an explanation. Since the discovery, a resolution is said to have been found in the Λ CDM model which employs the cosmological Λ term to help fit with the observational data.

We must stress once again that, in observational astronomy, the redshift is determined via the shift in *frequency* – not wavelength – of the emitted photons. This frequency-versus-wavelength distinction is significant. The scale-dependent c drastically alters the frequency-based redshift values as opposed to the wavelength-based ones. We must reexamine the observational data with this new insight.

With $\nu = c/\lambda$ and $c \propto a^{-\frac{1}{2}}$, the ν -based redshift acquires an extra scaling factor from c and obeys the modified Lemaitre formula (71):

$$1+z = a^{-\frac{3}{2}}(t_e)$$

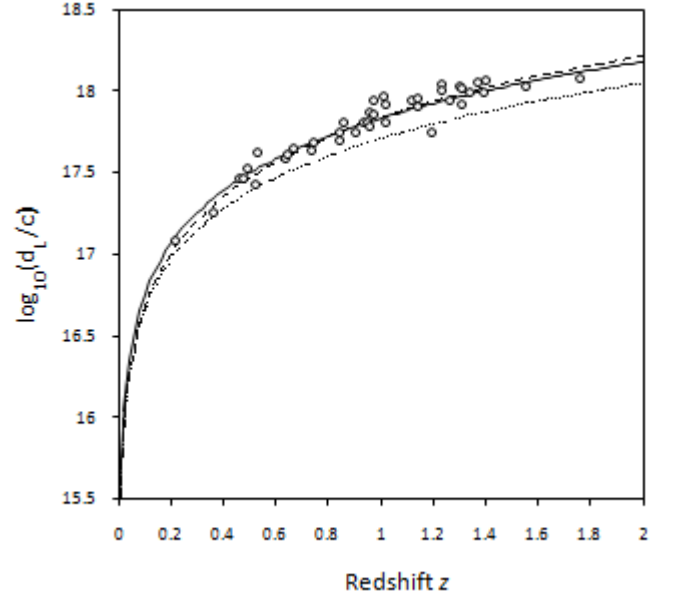


FIG. 1: Comparison of various luminosity distance-redshift formulae to Type Ia supernovae data. Open circles: 41 data points of high- z objects listed in Table 4 of Ref[13]. Long-dashed line: Λ CDM model's formula, Eq(100), with $H_0 = 70.5$, $\Omega_M = 0.27$, $\Omega_\Lambda = 0.73$, $\Omega_{curv} = 0$. Dotted line: Einstein-de Sitter model's formula, Eq(101), with $H_0 = 70.5$ ($\Omega_M = 1$, $\Omega_\Lambda = 0$, $\Omega_{curv} = 0$). Solid line: our model's formula, Eq(107), with $H_0 = 37.4$.

The λ -based redshift, on the other hand, obeys the traditional Lemaitre formula (63):

$$1+z = a^{-1}(t_e)$$

The ν -based redshift formula leads to a series of revisions, the culmination of which is the modified photometric distance-redshift relationship (107):

$$\frac{d_L}{c_0} = \frac{2}{3H_0} (1+z)^{\frac{5}{6}} \ln(1+z)$$

which plays the key role in our analysis of Type Ia supernovae in what follows.

Observational data are cited in the form of distance modulus $m - M$ which is related to photometric distance d_L as:

$$m - M = 5 \log_{10}(d_L/Mpc) + 25 \quad (108)$$

We retrieve the observational data of 41 high- z objects from Table 4 of [13]. The values of z and of $m - M$ are listed in columns 2 and 3 in the Table respectively. We then extract the photometric distances d_L via (108).

Figure 1 is the main result of this Section. The vertical axis measures $\log_{10}(d_L/c)$ where c is the value of speed of light measured on Earth (i.e., $300,000 \text{ km} \cdot \text{sec}^{-1}$). The set of 41 Type Ia supernovae data are shown in open circles. The solid line shows our model's Formula (107)

corresponding to one parameter $H_0 = 37.4$ that best fits to the data. The long-dashed line corresponds to Λ CDM model's Formula (100) with 3 parameters $H_0 = 70.5$, $\Omega_M = 0.27$, $\Omega_\Lambda = 0.73$ (besides $\Omega_{curv} = 1 - \Omega_M - \Omega_\Lambda = 0$). The dotted line shows Einstein-de Sitter universe's Formula (101) with $H_0 = 70.5$.

The mean average error (MAE) between a model's prediction and the 41 observational data is:

$$\frac{1}{41} \sum_{i=1}^{41} \left| \log_{10} \frac{d_L^{model}(i)}{c} - \log_{10} \frac{d_L^{observed}(i)}{c} \right|$$

The MAE of our Formula (107) is 0.0469 which is comparable to the MAE of Λ CDM model (100), 0.0476.

The most striking result is a new – and much lower – value for Hubble constant $H_0 = 37.4$. The reduction in value of H_0 (from the conventionally accepted 70.5) leads to two very important consequences:

1. A reduction in value of the critical density: $\rho_c = 3H_0^2/(8\pi G)$ is reduced by the factor $(70.5/37.4)^2 \approx 3.54$. That means the actual ρ_c should be only 0.28 time the value previously assumed. Interestingly, the new value of ρ_c approximately equals the total amount of baryonic matter and dark matter found in the universe. The budgetary shortfall in density (to maintain a near-flat space) *disappears*. “Dark energy” is not needed.
2. The universe age: Our universal age formula (53) $t_0 = 2/3H_0^{-1}$ yields 17.4 Gly, a universe sufficiently older than its oldest stars. The overestimated value of 70.5 for H_0 , on the other hand, led to a much younger age at 9.3 Gly if one assumes the Einstein-de Sitter model's age formula (which happens to be $t_0 = 2/3H_0^{-1}$ as well). Such a young universe would be at odd with the established age of the oldest stars. This formed the basis to reject the (flat) Einstein-de Sitter model and seek for other answers.

A standard answer is said to have been found in the Λ CDM model's age formula (54): $t_0 = (2 \tanh^{-1} \sqrt{\Omega_\Lambda}) / (3H_0 \sqrt{\Omega_\Lambda})$ which yields 13.8 Gly for $H_0 = 70.5$, $\Omega_M = 0.27$, $\Omega_\Lambda = 0.73$, $\Omega_{curv} = 0$. The Λ CDM model assumes a deceleration phase followed by an acceleration phase such that $t_0 \approx H_0^{-1}$, the Hubble time. In the light of our analysis which reveals a reduction in H_0 , the Λ CDM answer is no longer advantageous.

These two results will throw important light onto the mainstream interpretation of an “accelerating” universe discussed in the next section.

F. “Accelerating” universe: How strong were the evidences?

In the absence of the cosmological Λ term, the expansion is expected to be decelerating. It therefore came

as a major surprise when astronomers in 1998 concluded from Type Ia supernovae data that the universe appears to be expanding at an accelerating rate [12]. The conclusion of an “acceleration” was indirect; it was inferred as follows. In these observations, distant supernovae appear substantially fainter than what would have been expected for objects at the same redshift. This in turn indicates that the supernovae were farther than what the standard model would have predicted for their redshift. An explanation is that the space expansion at the time was slower than it is now, or, in other words, the expansion has been speeding up; hence the name “accelerating” expansion. This interpretation has become mainstream and fueled both observational and theoretical searches for a form of “dark energy” that supposedly speeds up the universe expansion.

In the light of several revisions that we have uncovered in this Section so far, the evidences for “acceleration” are not as solid as they seem. The foremost reason: *the interpretation relies on a false redshift formula!*

Let us first review the intuition behind the “acceleration” conclusion. For this purpose, consider the Hubble law which is a good first-order approximation for the distance-redshift relationship. (The Hubble law serves as a baseline. The higher-order effects will not affect the intuitive picture.) The farther the object, the higher its redshift. Per Hubble law, its redshift z and distance to Earth d are in linear proportion:

$$z = \frac{H}{c}d + \text{higher-ordered terms}$$

or

$$d \approx z \frac{c}{H} \quad (109)$$

where H is the rate of expansion at redshift z . The value of z is deduced from the photon's redshift. The value of d is deduced from the luminosity of the object.

At a given z , the supernova had been expected to be at a distance d dictated by Eq(109). What astronomers found [12] is that the (fainter than expected) supernovae were at a greater distance than given in (109).

How can this deviation be explained? There are actually *two* alternatives:

- (A) For a given z , the higher d than usual implies a *lower* H than usual. So, the supernovae must have experienced a lower rate of expansion H than now. The conclusion would be that space has been expanding faster and faster. The universe is thus accelerating.
- (B) For a given z , the higher d than usual implies a *higher* c than usual. So, light in the past must have traveled faster than it does now. The conclusion would be that light has been “slowing down”. The universe is not accelerating.

Scenario (A) is the standard “accelerating” universe interpretation. At the moment, it appears at least as plausible as scenario (B). However, we shall soon show a

deeper analysis that would weaken the case in support of scenario (A).

Scenario (B) accommodates the scale-dependent speed of light. The more distant the supernovae, the faster light was traveling when it left them, and thus the greater the distance light could cover to reach Earth. The supernovae were farther and thus appear dimmer.

We believe that scenario (B) is partly responsible for the “acceleration” found in Type Ia supernovae. Note that we deliberately tried to avoid the “light-slows-down” misnomer for scenario (B) since it fails to reflect the right physics. In our scale invariance consideration, light speed gets scaled as $a^{-\frac{1}{2}}$; as the universe expands, c becomes smaller. In effect, c “slows” down as the universe ages. But in principle, the distinction is conceptually important: c is reduced *not* as a result of cosmic time passage but as a result of the growth in the scale factor.

On the technical side, there is a serious problem in the way Type Ia supernovae data has been analyzed, a problem that would throw the resulting conclusion of an “accelerating” expansion (i.e., scenario(A)) into doubt. In those traditional studies, the redshift z was deduced from the wavelength-based formula which has been mistakenly assumed to be identical with the frequency-based formula. Once again, the two formulae are different and the correct one to be used is the ν -based formula (since astronomers measure photon’s frequency not its wavelength). In our critical analysis presented in the previous section, we utilized the modified luminosity-redshift relationship derived from the ν -based redshift. The modified relationship (107) is found to be *parsimoniously* in an agreement with high- z data as good as the traditional relationship (100) does, as shown in Fig.1.

There is yet another way to interpret the apparent acceleration. If one were able to directly compare the current speed of expansion $\dot{a}(t_o)$ with the speed of expansion when photons were emitted $\dot{a}(t_e)$, one would get (recall Eq(36), $a \propto t^{\frac{2}{3}}$):

$$\frac{\dot{a}(t_o)}{\dot{a}(t_e)} = \left(\frac{t_e}{t_o}\right)^{\frac{1}{3}} \quad (110)$$

which drops as the current time t_o proceeds. This means a deceleration. However, if one is only able to measure a speed of expansion *discounted* by the prevailing speed of light, one would instead get (note that $c \propto a^{-\frac{1}{2}} \propto t^{-\frac{1}{3}}$):

$$\frac{\dot{a}(t_o)/c(t_o)}{\dot{a}(t_e)/c(t_e)} = \frac{\dot{a}(t_o) c(t_e)}{\dot{a}(t_e) c(t_o)} = \left(\frac{t_e}{t_o}\right)^{\frac{1}{3}} \frac{c(t_e)}{c(t_o)} = \text{const} \quad (111)$$

which shows neither a deceleration nor an acceleration. So, the observation data thought to have indicated an “acceleration” are not as compelling as they seemed.

The reduction in H_0 uncovered in our analysis of Type Ia supernova (see Section V E) provides further insights:

1. With $H_0 = 37.4$, the universal age formula (53) neatly restores the age back up to 17.4 Gyr. The

standard Λ CDM explanation that assumes a deceleration phase followed by an acceleration phase is no longer advantageous. In addition, it is not as parsimonious as our formula which involves only one free parameter H_0 .

2. A new form of “dark energy” is often cited as a driving force for the “acceleration”. With $H_0 = 37.4$, the critical density is only 0.28 time the value traditionally thought. The known amount of baryonic and dark matters neatly fits into this budget (to achieve a flat space). There is no need to invent “dark energy” to make up a budgetary shortfall. In other words, “dark energy” is absent.

In conclusion, the observational and theoretical evidences said to be in support of an “accelerating” universe and subsequently the existence of “dark energy” are weaker than traditionally thought. The rationale comes in three accounts:

- Type Ia supernovae data have been routinely analyzed based on a problematic Friedmann model and a set of flawed distance-redshift relationships.
- The scale-invariant cosmological model offers a natural explanation to Type Ia supernovae (i.e., scenario (B)) and a solid performance for the data of high- z objects as shown in Section V E. In this model, Type Ia supernovae data are rather viewed as a *confirmation* of the scale-dependent speed of light.
- The “acceleration” interpretation often cites the budgetary shortfall and the age inconsistency – for which “dark energy” would be a convenient recipe – as supporting evidences. The absence of “dark energy” in our resolutions (1) and (2) for the budgetary and age problems thus further undermine the “accelerating” universe hypothesis.

We call for a thorough reexamination of the data to take account into the role of scale invariance which Friedmann’s model overlooks.

G. An open problem – the lensing effect for low-redshift objects

It is a tantalizing prospect that the scale dependence in speed of light is the underlying physics responsible for the “acceleration” effect observed in Type Ia supernovae. However, to build a compelling case in support of this possibility, full analyses for low- z objects are required.

It is important to note that our Milky Way-bound measurements are subject to gravitational confinement; the Milky Way resists the expansion of the universe. This fact is not a nuisance but rather a *necessity*. If our rulers and clocks were to “feel” space expansion, their length and rate would adjust accordingly to cancel out

any physical effects space expansion would have incurred. In particular, the redshift in lightwave emitted from distant galaxies would be undetectable. Due to gravitational confinement, light in our direct surrounding travels faster than that in an otherwise unbounded region, say, beyond the outskirts of the Milky Way. (The velocity of $300,000 \text{ km} \cdot \text{sec}^{-1}$ that we measure for light from Earth-bound source is *not* the value c_0 for light in the empty space.) This effect has two consequences:

- Refraction of light as it travels at varying speed: Detailed modeling of the mass distribution of the Milky Way (to specify the stress tensor in the Einstein equation) is an open – and crucial – problem. The stress tensor determines the *local* rate of expansion for the scale factor.
- Corrections in the measurement of distances for nearby objects: The value c_0 is currently undetermined. So far, to circumvent this problem, we deliberately expressed *distances in light years*. All distances are in the form of d_L/c_0 , d/c_0 and so forth (see Eqs(107, 72)). This procedure is effective for distant objects but may no longer be so for nearby objects.

In this preliminary report, we only hope to sketch a possible outcome once these issues are taken into account. After discounting the variable c (see Eq.(111) and the comment that followed) the rate of expansion should show neither a deceleration nor an acceleration. Type Ia supernovae data, on the other hand, do reveal some (albeit apparent) “acceleration” in their distance-redshift plot. How could this residual “acceleration” be accounted for? In addition, it has been argued that the “acceleration” phase was preceded by a deceleration phase; a transition takes place at around $z \sim 0.5$ [14]. We speculate that both effects are a result of a crossover in play: low- z objects obey more closely the traditional distance-redshift relationship (that is to say, they are less susceptible to the variable c effect and their Hubble value is thus closer to 71). On the other hand, high- z objects follow the modified distance-redshift relationship (namely, a lower Hubble value). As we gradually move from low z to high z , there might be a crossover between a high Hubble value to a lower one. That would result in an “upward” slope for the curve on the distance-versus-redshift plot. This issue is an open problem which requires detailed modeling of the refraction effect mentioned above.

VI. IMPLICATIONS IN COSMOLOGY

This section devotes to resolving a series of theoretical problems encountered in modern cosmology. Attempts to reconcile these problems with the Big Bang model often resort to the inflationary universe hypothesis and the cosmological constant. We shall seek a consistent treatment to the problems from the modified Friedmann equations.

A. Resolution to the “runaway density parameter” problem

Let us first review the situation within the traditional framework. From the traditional Friedmann equation (B26) and the critical density defined as $\rho_c = 3H^2/(8\pi G)$, the density parameter

$$\Omega \equiv \frac{\rho}{\rho_c} \quad (112)$$

satisfies

$$(\Omega^{-1} - 1) \rho a^2 = -\frac{3kc^2}{8\pi G} \quad (113)$$

The RHS is a constant, whereas ρa^2 scales as a^{-1} . To compensate the drop in ρa^2 as a grows, $|\Omega^{-1} - 1|$ must grow and Ω is driven away from 1 if it started away from 1. This spells the instability trouble for Ω . The runaway of Ω from 1 has been known to be rapid. In order to account for a relatively flat space at our current era, the universe must have started out extremely flat and the value for Ω at recombination time must have been fine-tuned to unity at high precision. The “runaway” problem posed a major challenge to the Big Bang theory.

In our model, this problem remarkably *disappears*. Recall that the modified Friedmann equation reads:

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3} \rho - \frac{kc_0^2}{a^3} \quad (114)$$

By rearranging the terms, we get:

$$\frac{\rho_c}{\rho} - 1 = -\frac{3kc_0^2}{8\pi G} \frac{1}{\rho a^3} = -\frac{3kc_0^2}{8\pi G} \frac{1}{\rho_0} \quad (115)$$

The RHS is a constant, so $\Omega \equiv \frac{\rho}{\rho_c}$ is a marginal variable, unchanged throughout the evolution of the universe! The dreadful fine-tuning issue disappears.

In hindsight, the reason behind the “runaway” problem in the traditional framework was the scaling mismatch raised in our Motivation section: The three terms in Friedmann’s equation, $(\dot{a}/a)^2 = 8\pi G/3\rho - kc^2/a^2$, scale differently. The scaling mismatch got transferred to Eq(113), creating the dreaded instability problem. In a sense, we could have made a “fix up” by allowing c^2 to decrease as a^{-1} to help cancel out the undesirable decrease in ρa^2 . But this heuristics ends here; the bandage quick fix is not logically consistent. The scale-dependency of c must be recognized *ab initio* as is done in (114) and (115).

To our knowledge, this is the first time a thorough resolution to the “runaway density parameter” problem naturally arises within Einstein’s theory of general relativity (without the extra assumptions of the cosmological constant and the inflationary universe hypothesis). Intimately related to the “runaway” problem, left unresolved, is the oldness problem. A universe started slightly away from criticality would have either succumbed to a

Big Crunch or quickly diluted for stars to have formed. Together with the “runaway” problem, the oldness problem is naturally resolved in our model.

The inflationary hypothesis [10] was put forward in the early eighties as a resolution to other problems (horizon, flatness, monopole) in the Big Bang cosmology and has developed into a mainstream body of research. The literature that covers this subject is extensive; here we only wish to outline a few relevant facts. The universe is said to have undergone an exponential growth in its very early stage, a growth which drove the density toward criticality, flattened space, and diluted the hypothesized monopoles. The force responsible for the outgrowth is assumed to be in the form of the cosmological Λ term that Einstein introduced and later abandoned. As is well documented, the Λ term inserts its own share of problems: (a) The coincidence that Ω_Λ happens to be of order 1 (~ 0.73 , and comparable to Ω_M) at our current epoch is a challenging question, given that ρ_Λ stays unchanged and ρ_M falls as space expands; (b) Its physical origin as vacuum energy yields a value of several orders of magnitude higher than the observational value. The inflationary hypothesis, in itself, requires fine-tuning in the form of the slow-rolling conditions and likely contains other logical issues [11].

In addition, the “runaway density parameter” difficulty continues to lurk inside the inflationary universe approach. The natural disappearance of the “runaway” problem in our model is thus a remarkable and attractive feature.

B. Resolution to the matter budget shortfall

In Friedmann’s model, the critical density defined as $\rho_c = 3H^2/(8\pi G)$ corresponds to a flat universe. It is an unstable fixed point for the density: a (closed) universe started out denser than the prevailing ρ_c would become denser and finally collapse in a Big Crunch, whereas an (open) universe started out less dense than ρ_c would become more diluted and expand faster than the critical growth rate into a Big Chill.

In our model, ρ_c continues to be the separatrix between closed and open universes. The density ρ determines the shape of the universe. However, the density plays no role in the fate of the universe; all three shapes expand in the critical fashion: $a \propto t^{\frac{1}{3}}$. For this reason, we prefer to call ρ_c the *flat-space density* in place of the critical density.

As pointed out in Section VE, the value of ρ_c has been *inadvertently overestimated*. Its actual value should stand at 0.28 time the value previously thought. The budgetary shortfall in density *disappears*. The baryonic and dark matter content is solely responsible for the flatness of space. “Dark energy” is absent.

C. Resolution to the age problem

The resolution to the universe’s age has been discussed in details in Sections VE and VF. Here, we only wish to recap. The age formula (53)

$$t_0 = \frac{2}{3H_0}$$

is robust. It is derivable solely from the scaling property of time (see Section III E). Furthermore, it is valid for all shapes of the universe.

With the corrected $H_0 = 37.4$, the universe age is $t_0 = 17.4 \text{ Gy}$ comfortably accommodating its oldest stars. The standard Λ CDM formula that invokes a deceleration phase followed by an acceleration phase in the universe expansion is no longer advantageous or necessary.

D. The past and future of the universe

Regardless of its shape (i.e., density), the universe always grows in accordance with a universal law: $a \propto t^{\frac{2}{3}}$. Once again, this law robustly arises from the scaling of time, $t \propto a^{\frac{3}{2}}$. Detailed structure of the evolution equation (i.e., the modified RW metric) plays little role here.

Likewise, regardless of its shape (and density), the universe will always expand in a critical fashion. The universe as a whole will not collapse into a Big Crunch. Represented in conformal time and comoving coordinates, the universe *always* looks the same on the large scale. It is an “eternal” universe.

The robustness of its evolution offers an alternative answer in place of anthropic arguments which are often invoked to explain why the universe reaches its current state. Assuming the sensitivity to its initial condition (the “runaway” problem and several other cosmic coincidences), there is often said to exist a vast number of universes – a multiverse – each of which began with different conditions. The end result is that we exist in one of the universes that could harbor intelligent life.

Anthropic arguments are no longer necessary in our approach. Leaving quantum uncertainties aside, the formations of stars, galaxies, life, and intelligence were largely deterministic. Their existences are not likely a matter of happenstance.

E. Resolution to the horizon problem

The cosmic microwave background (CMB) observational result shows a very uniform distribution (to the accuracy of 10^{-5}) of cosmic radiation across the horizon. This uniformity presents a serious challenge to the Big Bang theory. It needs to reconcile the observed uniformity with the fact that the current horizon is not causally connected. The inflationary universe hypothesis was partially motivated to explain away this problem [10]. Before the inflationary phase, the whole horizon must have

been in causal contact and thus reached a thermal equilibrium. As the inflation kicked in, different sections in the horizon were rapidly pulled away from one another and, as a result, now appear separated.

An alternative explanation to the horizon problem was originated by Moffat via variable speed of light [4]. If the velocity of light in the baby universe was higher than it is now by a factor of 10^{30} , the horizon of the early universe would be in causal contact and thus achieve uniformity. Moffat's contributions to this direction are well documented, and it is beyond our expertise to reproduce his results here. Beside his original proposal that c underwent a first-order phase transition from a very high value to its current value, subsequent developments of Moffat, and of Magueijo and other researchers [5, 6] also allowed a smooth variation in c .

Our treatment allows c to vary with the scale factor and, otherwise, is in line with Moffat and Magueijo's insights in resolving the horizon problem. More concretely, in the modified RW metric, the cosmological horizon is given by:

$$l_H(t_0) = \int d\eta = \int_0^{t_0} \frac{c(t_0)d\tau}{a^{\frac{3}{2}}(\tau)} \quad (116)$$

Since $a(\tau) = \left(\frac{\tau}{t_0}\right)^{\frac{2}{3}}$, we get

$$l_H(t_0) = \int_0^{t_0} \frac{c(t_0)d\tau}{\tau/t_0} = c(t_0)t_0 \int_0^{t_0} \frac{d\tau}{\tau} = \infty \quad (117)$$

The cosmological horizon is thus (logarithmically) divergent! This result neatly helps explain the near uniformity in our current horizon.

The new elements in our approach are:

- The explanation for variable c is natural. It comes from the scale invariance requirement. Unlike in other existing approaches, the need to devise a dynamics for variable c disappears.
- That the divergence in (117) is logarithmic, thus *parsimonious*, is a curious result. It holds for a general d -dimensional space. From Appendix A, c scales as $a^{1-\frac{d}{2}}$, thereby the conformal time $\eta \propto a^{-\frac{d}{2}}$. Since $t \propto a^{\frac{d}{2}}$, $a \propto t^{\frac{2}{d}}$. So, the cosmological horizon is:

$$\begin{aligned} l_H(t_0) &= \int d\eta = \int_0^{t_0} \frac{c(t_0)d\tau}{a^{\frac{d}{2}}(\tau)} \\ &= c(t_0)t_0 \int_0^{t_0} \frac{d\tau}{\tau} = \infty \end{aligned}$$

which is also divergent *logarithmically*.

F. The flatness problem

The Wilkinson Microwave Anisotropy Probe (WMAP) has confirmed that the universe is flat within 0.5% margin of error.

In the traditional Big Bang paradigm, flatness is a vital condition for the universe to survive and for its stars to have formed. The universe thus required specific mechanisms to achieve and sustain flatness. The inflationary scenario offered one such mechanism; a sudden and powerful spurt in the early universe driven by the Λ term would flatten it regardless of its starting condition.

In our model, on the contrary, flatness is a curious – but not critical – feature. Flatness is not a difficulty per se, nor does it play any role. Even if it started out away from flatness, the large-scale universe would trace an identical course of history (see Section VID). The observed flatness is nonetheless a curiosity, a solution to which likely lies outside the scope of general relativity (and our cosmological model) since general relativity is a theory more about the universe evolution than about its origin.

G. Uniqueness in scaling of density and pressure

In Friedmann's model, the equation of state is

$$\dot{\rho} = -3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) \quad (118)$$

in which c is a constant. The standard practice assumes different forms of density and pressure on the case by case basis. Most notable are:

- Non-relativistic matter (or pressureless “dust”): $p = 0$, leading to $\dot{\rho}/\rho = -3\dot{a}/a$ or $\rho \propto a^{-3}$.
- Radiation: Since the electromagnetic stress-energy tensor is traceless, one deduces that $p = \frac{1}{3}\rho c^2$, leading to $\dot{\rho}/\rho = -4\dot{a}/a$ or $\rho \propto a^{-4}$.
- Cosmological Λ term: $p = -\rho c^2$, leading to $\rho = \text{const.}$

In the light of scale invariance, this practice is *questionable*, however. As space expands, if different forms of matter obey different scaling rules, one would be able to devise an experiment to detect the *local* space expansion. This would be in violation of the scale invariance principle which demands that no test of the laws of physics can in any way distinguish between reference scales. So long as we assume the perfect fluid content for the Einstein field equation, there is *one and only one* admissible set of scaling rules for ρ and p :

$$\begin{aligned} \rho &\propto a^{-3} \\ p &\propto a^{-4} \end{aligned}$$

The role of scale invariance has been overlooked in Friedmann's model. Without the scale invariance requirement to cross-check, the model's discrimination among different forms of matter has gone unchallenged. We believe that the standard classification of radiation versus

matter together with the separation between radiation-dominated era and the matter-dominated era should be abandoned.

There is a puzzling feature for the pressure term in our model, nonetheless. The pressure is negative, per Eq(41). On the other hand, the stress-energy tensor of radiation is known to be traceless which implies that

$$p = \frac{1}{3}\rho c^2 \quad (119)$$

in contrast with (41). This disagreement could indicate that radiation is not a form of perfect fluid to be used in Einstein's field equation.

H. Outstanding problems in our model

In the Big Bang model, the universe was dense and hot in the past. As it cooled down during expansion, it went through a series of “events”. In one of such events, when its temperature dropped to about 3,000 Kelvin, which is said to have taken place about 380,000 years after the Big Bang, electrons began to couple with protons to form hydrogen atoms. The less energetic radiation which suffered “Doppler theft” was no longer able to ionize the atoms. This is called the recombination event.

One pressing challenge in our model is to explain the recombination event. In the “eternal” universe in which all forms of energy scale the same way, $E \propto a^{-1}$, photon's energy and the binding energy of the hydrogen atom maintained a fixed ratio. How then, in theory, electrons could elude the energetic photons and combine with protons?

In this preliminary study, we only hope to qualitatively outline a possible account for the recombination event. The answer may be found in the inhomogeneity of the local scale factor. In the cosmic background, quantum or thermal fluctuations would initially form regions with slightly higher density than average. Due to the attraction nature of gravity, the denser regions would attract further mass and gradually grow in magnitude and size. An inhomogeneity emerged in the local scale. The seeded regions became more and more gravitationally bounded and began to resist the space expansion such that the energy of matter in these regions stopped falling despite the overall expansion of the universe. Once the gravitational confinement was sufficiently developed, the red-shifted photons in the cosmic background would no longer be energetic enough to ionize the atoms that found coziness in the seeded regions.

Other outstanding problems in our model include the treatments of detailed CMB map observed in WMAP, baryogenesis, nucleosynthesis. These subjects are beyond the current scope of our preliminary study. We hope to tackle these problems in future research.

VII. IMPLICATIONS IN PHYSICS

A. Non-conservation of energy and momentum

Energy and momentum are “constants” of motion. That they are conserved quantities is the result of Nöther's theorem applied to the translational symmetries of space and time.

In our model, energy and momentum are no longer conserved. As discussed in Section II C, energy of *every* system scales as a^{-1} . This effect has already been known for radiation; the standard interpretation has been that photons suffer energy loss due to the “Doppler theft”. On the other hand, the standard interpretation exempts other forms of matter, such as baryonic matter, from the energy loss effect. This treatment is inconsistent for two reasons:

- The binding energy in atoms is also of electromagnetic nature. How, then, does this form of energy manage to escape the “Doppler theft”? The fact that galaxies are in gravitationally bounded regions is irrelevant. Extragalactic regions are not empty; one can always imagine an atom in one of these regions where the redshifted lightwave happened to pass by.
- Radiation is singled out for special treatment. Scale invariance demands that *all* physical phenomena transform the same way upon a scale change. Baryonic matter must also transform like radiation; otherwise one would be able to detect the scale change using radiation and atoms within one's local lab.

In the expanding universe the effect of energy “loss” (and thus “non-conservation” of energy) arises when one compares the energy values at different spatial scales. The violation of energy conservation is not due to the lack of time translational symmetry so that Nöther's theorem fails to apply. Momentum, scaling as $a^{-\frac{1}{2}}$, is not “conserved” either. Einstein's field equation indeed confirms these results. Applying Bianchi's identity to Einstein's field equation leads to:

$$\nabla^\mu \left(\frac{1}{c^4(a(x))} T_{\mu\nu} \right) = 0 \quad (120)$$

which is obviously not equivalent to the usual conservation law:

$$\nabla^\mu T_{\mu\nu} = 0 \quad (121)$$

Furthermore, if the scale factor can vary in space, even the notion of total energy or momentum of a closed system loses its meaning. We therefore view the necessity to revise Nöther's theorem a moot point.

How does the lack of energy conservation affect the formation of atoms and stars in the early universe? Since all forms of energy scale identically, the decay rates of radioactive isotopes – an effect of quantum tunneling

through potential barriers – were exactly as they are now. Likewise, since all binding energies scale the same way, chemical bonds between atoms in molecules were unaffected. Energy scaling thus incurred no effects on nuclear physics and chemistry in the early universe.

B. Mach’s principle

In [15], Bondi and Samuel discussed several versions of Mach’s principle. Among them, Mach8 is of particular interest to us. It states that: “ $\Omega = 4\pi\rho GT^2$ is a definite number of order unity”. Here, T is the Hubble time. (Note that their definition of Ω is not the density parameter $\frac{\rho}{\rho_c}$ per (112).) It was observed in [15] that “ Ω does seem to be of order unity in our present universe, but ... only the Einstein-de Sitter (universe) makes this number a constant, if Ω is not *exactly* one”. The authors further commented: “Making a theory in which this approximate equality appears natural is a worthwhile and ongoing effort (eg inflationary cosmologies)”. It is noteworthy that the correspondence $\Omega \approx 1$ is yet *another cosmic coincidence* that begs a natural answer.

In our cosmological model, taking the Hubble time as $T = H^{-1}$, Bondi-Samuel’s Ω is recast as

$$\begin{aligned}\Omega &= 4\pi G\rho H^{-2} \\ &= 4\pi G\rho_0 a^{-3} H_0^{-2} a^3 \\ &= 4\pi G\rho_0 H_0^{-2} \\ &= \frac{3}{2} \frac{\rho}{\rho_c}\end{aligned}$$

which *is* indeed a constant since $\frac{\rho}{\rho_c}$ is a constant per Eq(115). Therefore, the fact that Ω is of order unity at our current time is no coincidence. It holds for every epoch. Interestingly, this equality robustly holds for all shapes of the universe (open/flat/closed) as well. It should *not* exactly equal one, but is determined by the density parameter, a result that was not *a priori* obvious nor anticipated in [15]. It is fair to say that Bondi-Samuel’s expectation of a worthwhile and natural theory to account for the behavior of Ω is effortlessly fulfilled in our model.

Mach’s principle posits that the inertial mass of an object is determined by mass distribution, both nearby and far away. This point has been under intense physical and philosophical debates since its inception, and it lies beyond our expertise to discuss this issue. Nevertheless, curiously enough, we observe that Mach’s principle apparently undergoes a *revival in a different form*: The values of some physical constants, most notably c and \hbar , measured at a particular spacetime point, do depend on the local scale of the vicinity of that point. The local scale, in turn, is expected to be determined by the mass distribution, both nearby or far away, in the universe.

C. A peculiar scaling rule for time

The Minkowskian geometry has unified space and time into a single entity, the 4-dimensional spacetime, in which they partake in equal footing. This desirable symmetry between space and time, curiously enough, appears to be broken in our scale invariance consideration: whereas space scales as a , time scales differently as $a^{\frac{3}{2}}$.

We must note that the peculiar scaling rule for time is not exclusive to our model. The scaling property for time, $t \propto a^{\frac{3}{2}}$, inevitably leads to the time evolution of the scale factor, Eq(36): $a \propto t^{\frac{2}{3}}$ which is the solution to the modified Friedmann equations. Interestingly, this particular time evolution of the scale factor also held for Einstein-de Sitter universe. So, in hindsight, the peculiarity in the scaling of time was already present in the traditional cosmological paradigm!

This fact helps allay the suspicions that the curious scaling rule for time is problematic and that scale invariance was the party of guilt. On the contrary, scale invariance helps uncover the asymmetric roles between space and time.

D. Open problems and speculations regarding our model in physics

This section is a collection of conceptual issues that we could envisage at the moment. It contains our speculations about potential implications of scale invariance in the context of theoretical physics. At this stage, they are entirely speculative. We do not know the answers to these questions; neither do we know whether the search for their answers would refute scale invariance and our cosmological model. We therefore feel the merit to include them in this report.

1. Local scale invariance, topological properties, and quantum dynamics?

Scale invariance consideration opens a very interesting possibility. Imagine that the spacetime manifold undergoes a *local* deformation – actual or fictitious – upon which its topological properties are unchanged but its geometrical properties get altered. Given that \hbar and c can adjust to “offset” the effect of the local scale change, the deformation can thus be “gauged” away. Would this “gauging” correspond to some new physics?

Should history be prolog, the $U(1)$ gauge symmetry in QED is a noteworthy example. In quantum mechanics, the phase of the wavefunction of electron at first was thought to have no physical reality; only the squared amplitude of the wavefunction is measurable in the form of probability. Yet a *local* transformation of the phase does alter the Lagrangian of a non-interacting electron. In order to restore the local gauge symmetry to the Lagrangian, it is necessary to couple the electron with a vec-

tor field A^μ that satisfies Maxwell's equations. When the electron wavefunction acquires a local phase shift, the vector field would transform accordingly to remove the impact of the phase change on the Lagrangian. The Yang-Mills extensions to non-Abelian $SU(2)$ and $SU(3)$ groups have led to spectacular successes in the electroweak theory and QCD.

It is natural to ponder if (and how) this very interesting scenario would repeat for scale invariance. Whether scale invariance has any implications in the dynamics of microscopic particles (in particular, whether Heisenberg's uncertainty principle could be the end result of such a space deformation getting "gauged" away by variations in \hbar and c yet, on average, still leaving some footprint) is a purely speculative but tantalizing possibility.

2. A possibility of superluminality?

If the scale change is not uniform in space and given that c varies with respect to scale, light might travel with different speeds along a null geodesic. The end result could be that a light signal might appear to travel superluminally if it is initially emitted and is finally detected in regions that correspond to lower light speeds. Whether this scenario takes place in reality is another highly speculative idea for the time being (and it would be far-fetched to speculate on its relevance to a recent announcement of superluminal neutrinos at OPERA [21].)

3. A peculiar scaling rule for boson

Recall the scaling rules (20-24), the boson field φ and fermion field ψ scale as

$$\varphi \propto a^{-1} \varphi \quad (122)$$

$$\psi \propto a^{-\frac{3}{2}} \psi \quad (123)$$

The total numbers of bosons n_B and fermions n_F scale as

$$n_B = \int d^3x |\varphi|^2 \propto a^3 (a^{-1})^2 \propto a \quad (124)$$

$$n_F = \int d^3x \bar{\psi} \psi \propto a^3 \left(a^{-\frac{3}{2}}\right)^2 = \text{const} \quad (125)$$

Upon a space expansion, the fermion count is unchanged. On the contrary, bosons proliferate but their density remains fixed. At the moment it is not clear to us the meaning and implications in the growth of the boson count.

4. The diminishing role of quantum laws in the baby universe?

As all scales in our model vary in proportion with one another, the Planck length was smaller in this exact proportion in the early universe. This is evident from the

scaling rules $\hbar \propto a^{\frac{1}{2}}$, $c \propto a^{-\frac{1}{2}}$, $G = \text{const}$ which leads to the scaling rule for the Planck length

$$l_P = \sqrt{\frac{\hbar G}{c^3}} \propto a \quad (126)$$

The separation in scales is preserved: the macroscopic scale (including those at electroweak unification) and the quantum gravitational scale do not mix. Likewise, the smaller (i.e., earlier) universe, the smaller the Planck constant and thereby the quantum effects. The conclusion is that quantum gravity was likely not a dominant agent in our baby universe. In [9] Casado raised this possibility which he pretends could help avoid the confrontation of general relativity and quantum physics concerning the Planck era. We believe that the quest to unify general relativity and quantum laws, as is being actively pursued in loop quantum gravity and in string theory, remains *conceptually* legitimate and profound. But at the practical level, the role of quantum physics in the baby universe was not as important as traditionally assumed.

VIII. COMPARISON OF OUR WORK TO THE EXISTING LITERATURE OF VARIABLE SPEED OF LIGHT IN COSMOLOGY

Prior to this work of ours, the speculation of variable speed of light (VSL) has had a record dated back to the early nineties. Moffat and subsequently Albrecht and Magueijo were the first to postulate the variation of the speed of light in the early universe [4, 5]. Their motivations were to explain away the horizon problem as an alternatives to the inflationary hypothesis. In the baby universe, a very large value of c about 30 orders of magnitude higher than its current value could account for the thermal equilibrium across different parts of the horizon [4]. Moffat initially proposed a mechanism for c to have undergone a first-order phase transition from such a high value to the value observed today [4]. Magueijo and Smolin hypothesized a dependency of c on the energy of the photon [6]. We believe that their pioneering ideas have encouraged the subsequent pursuits in the VSL field.

Nevertheless, there are several fundamental distinctions between their approaches and ours. First, VSL theories were classified in two categories: "*those where c varies in space-time, and those where it varies with the energy scale*"[7]. Our approach belongs to neither category; c varies with the spatial scale. The $\dot{c}(t)$ term or the like is absent. Second, it was asserted that "*all theories under the name of VSL break Lorentz invariance in some way*"[7]. As was discussed in Section IV, we forsake the notion that Lorentz invariance should hold globally in a curved spacetime; nevertheless our treatment does respect general relativistic invariance. Indeed, it also respects Lorentz invariance *locally*. Third, we point out the conceptual shortcomings of Friedmann's model and the necessity to improve it. It is the logical basis that motivated us, besides the need to resolve observational and

theoretical issues. We utilize the correspondence procedure to generalize the fundamental laws of physics such that they are valid in the expanding universe; we also find the replacement to the laws that do need modify, such as Friedmann's equations and the conservation laws. We provide a concrete mathematical model to do calculations, both for the results presented in this report and for future analyses in the field. Last but not least, the most crucial point is: our approach justifies not only that c can vary or should vary. It *must* vary.

Other speculative ideas available in the public domain exploited some “coincidental” equalities between the speed of light and the “size” or age of the universe. These *ad hoc* relationships are false because they miss out information about the density parameter Ω which must be present per our discussion of Mach's principle (Section VII B), and are fortuitous as a matter of one's arbitrary definition of the universe's “size”. There is no logical qualification as of why and how c was related to the size or age of the universe at large. A viable mathematical framework or justifications on the validity of established physical laws in a world with variable c are all lacking.

Amongst these efforts, the proposal advanced the most along our treatment belongs to Casado [9]. His numerological guesswork about the dependency of \hbar on the universe's “size” triggered our interest to seek a deeper logical foundation and construct a coherent answer to the problem.

IX. CONCLUSIONS

We offered a coherent answer to a host of problems in modern cosmology. Our unconventional solution is achieved via an abandonment of the universality in the fundamental constants of Nature. The logics in our treatment arose naturally as follows:

- Physical laws are required to be invariant under scale transformation. The Planck constant and the speed of light need to adjust accordingly as functions of the local scale factor in order to keep physical laws invariant. The scaling properties for \hbar and c are: $\hbar \propto a^{\frac{1}{2}}$, $c \propto a^{-\frac{1}{2}}$. In the expanding universe, the variations of \hbar and c arise from the growth in the (local) scale factor; they are not a result of the cosmic time passage. See Section II.
- The farther a cosmological object, *the lower value for \hbar and a higher value for c that the object experienced*. We must adjust \hbar and c accordingly for the object. This fact has been overlooked in the standard paradigm of cosmology.
- Friedmann's model violates scale invariance and neglects the scale dependency in c . The mismatch in scaling of terms in Friedmann's equations is the root of the “instable” or “runaway” density parameter problem as well as an array of other fine-tuning

problems encountered in the Big Bang theory. We abandoned Friedmann's equations and sought an alternative.

- We modified the Robertson-Walker metric to include the scale dependency for c , and further imposed the scaling rule of c on the $8\pi G/c^4$ coefficient and the stress-energy tensor $T_{\mu\nu}$ in the Einstein field equation. We then derived the modified Friedmann equations which are fully scale invariant. See Section III.
- An exposition on the compatibility of nonuniversal speed of light with Lorentz invariance is provided in Section IV. We emphasized the distinction between the two following concepts: constancy versus universality. Whereas c is constant in all reference frames, it needs not be universal across spacetime. We also justified the soundness of our approach in comparison with Ellis's criteria [8].

Our cosmological model was next applied to resolve a series of observational issues in cosmography in Section V. The scaling property of c leads to a series of major revisions. Of the central role is a modification in Lemaitre's redshift formula. Its ramifications are sweeping:

- Modifications in the Hubble law and the Hubble constant. The traditional Hubble law was missing a multiplicative factor of 3/2 in its coefficient. The Hubble constant has thus been inadvertently overestimated.
- Modifications in the distance-redshift and luminosity-redshift relationships.
- We carried out a critical reexamination of Type Ia supernovae data, from which a corrected value of $H_0 = 37.4$ is obtained for the Hubble constant. See Section V E.
- With $H_0 = 37.4$ instead of 70.5, the critical density is only 0.28 time the value traditionally assumed. The baryonic and dark matter content is sufficient for the density budget (to make the space flat). The budgetary shortfall problem disappears. “Dark energy” is absent.
- With $H_0 = 37.4$, the universe age is restored back up to 17.4 Gly via the universal formula $t_0 = 2/3H_0^{-1}$. This resolves the age problem while bypassing the Λ CDM deceleration-acceleration scenario.
- Photons emitted from distant Type Ia supernovae initially traveled at higher speed and thus covered a greater distance. The supernovae were farther and thus appeared fainter. We worked out a detailed reasoning which points toward an alternative explanation: the universe is not “accelerating” but is a result of physical laws, including c and \hbar , adapting to new scale as the universe expands. Further

works on low-redshift objects are needed to verify or refute this possibility. See Sections V F and V G.

The prospect that Type Ia supernovae reveal a footprint of the scale-dependent c (and \hbar), in our view, would offer a far more fascinating and fertile playground than the “acceleration” scenario does because it would open up multiple doors into new research territories.

In Section VI we applied our model to resolve a host of logical and theoretical difficulties that often find remedies in the inflationary hypothesis and the cosmological constant. In particular:

- The “runaway density parameter” problem naturally disappears. Unlike Friedmann’s equations, the correct scaling property is restored in our modified Friedmann equation. This point is crucial to help remove the instability of the density parameter. We consider this resolution to be one of the most appealing features of our model. See Section VIA.
- The density shortfall problem disappears, thanks to the corrected Hubble value $H_0 = 37.4$. There is no need to invent “dark energy” to make up the budget.
- The age problem disappears, because of $H_0 = 37.4$. We obtain a new age figure at 17.4 Gy . The Λ CDM deceleration-acceleration account for age is not necessary.
- The horizon problem is resolved thanks to a high light speed in the early universe. The cosmological horizon is shown to be logarithmically divergent.
- The history and future of the universe are largely deterministic. Anthropic arguments are avoided. The universe is “eternal”; on the large scale, it always appears the same.

The role of scale invariance has also been overlooked in physics. If scale invariance proves to be a principle of Nature, it likely will deepen our understanding of the structure of spacetime in general relativity and open up a new paradigm – a nonuniversality in the fundamental constants of Nature. Like relativity principle which powerfully constrains the admissible structure of physical laws – the laws must be in a covariant form, scale invariance powerfully constrains the admissible form of physical laws – the constants of Nature must adapt to the local scale.

The appeal of our approach is in its simplicity. We showed how the structure of known fundamental laws is fully retained, and via the correspondence procedure (see Section IID) how they are readily extendible to describe physics in the expanding universe and the physics of the expanding universe. The “new physics” emerging from our approach is not in an exotic form of matter or a speculative type of interaction; nor is it an overthrow of the

“old” establishment, such as Einstein’s theory of gravitation. Rather, it is a *new perspective* to view known physical laws in the evolutionary spacetime. It leads to a host of rather surprising implications. Most notable are the violation of conservation in energy and momentum, the revival of Mach’s principle, the asymmetry between space and time. In addition, quantum laws should be affected due to the nonuniversal \hbar . We also raised a number of speculative proposals such as a possibility of superluminality and a potential connection between scale transformation and quantum dynamics.

Several outstanding problems remain. The lensing effect for low-redshift objects, the treatments of CMB map detected in WMAP, the recombination era, baryogenesis, nucleosynthesis are a few that merit future attention.

Our exploration of scale invariance and its implications in cosmology and physics is cursory in several parts. We hope that the interested reader will help us understand its many pressing conceptual and practical issues.

Acknowledgments

It is unusual to acknowledge the role of a book, rather than of a human fellow, in a research report. Yet after all, good books are written by good man, especially by the masters who – through their selfless and artful writing – taught this author about the subject and helped shape up his thinking. Amongst the countless authors we are privileged to have learnt from, Rovelli [19] provided the clarity regarding the utilization of invariance principles in Einstein’s theories – an idea that directly inspired and guided us in this piece of work.

Appendix A: Scaling rules for the Planck constant and speed of light in d -dimensional space

The partition function of QED (25) in d spatial dimensions (with $d^d x$ in place of $d\vec{x}$) is invariant under the scaling rules:

$$\begin{aligned}\vec{x} &\rightarrow a\vec{x} \\ t &\rightarrow a^{\frac{d}{2}}t \\ \hbar &\rightarrow a^{2-\frac{d}{2}}\hbar \\ c &\rightarrow a^{1-\frac{d}{2}}c \\ e &\rightarrow e \\ m &\rightarrow m\end{aligned}\tag{A1}$$

It is easy to check that the above scaling rules leave unchanged the Schrödinger equation of the hydrogen atom in d spatial dimensions:

$$i\hbar\frac{\partial}{\partial t}\Psi = \left[-\frac{\hbar^2}{2m}\nabla^2 - \frac{e^2}{r^{d-2}}\right]\Psi$$

Appendix B: Derivation of the modified Friedmann equations

It proves more convenient to work with the radius of the universe than with its scale factor. We shall recover the scale factor at the end of our derivation. In our calculation until Eqs(B18, B19) we closely follow Ref[18] and adopt its choice of coordinates.

Let us first consider the closed universe case, $\kappa = 1$. The modified RW metric reads:

$$ds^2 = c_0^2 \frac{R_0}{R(t)} dt^2 - R^2(t) (d\chi^2 + \sin^2 \chi d\Omega^2) \quad (\text{B1})$$

$$= R^2(\eta) [d\eta^2 - (d\chi^2 + \sin^2 \chi d\Omega^2)] \quad (\text{B2})$$

in which $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$ and η is the conformal time defined via:

$$d\eta = c_0 R_0^{\frac{1}{2}} \frac{dt}{R^{\frac{3}{2}}(t)} \quad (\text{B3})$$

The variables $(x^0, x^1, x^2, x^3) = (\eta, \chi, \theta, \phi)$ are of dimensionless unit. The metric tensor is diagonal:

$$\begin{aligned} g_{00} &= R^2(\eta) \\ g_{11} &= -R^2(\eta) \\ g_{22} &= -R^2(\eta) \sin^2 \chi \\ g_{33} &= -R^2(\eta) \sin^2 \chi \sin^2 \theta \end{aligned}$$

So is its contravariant counterpart:

$$\begin{aligned} g^{00} &= R^{-2}(\eta) \\ g^{11} &= -R^{-2}(\eta) \\ g^{22} &= -R^{-2}(\eta) \sin^{-2} \chi \\ g^{33} &= -R^{-2}(\eta) \sin^{-2} \chi \sin^{-2} \theta \end{aligned}$$

Of the 64 Christoffel symbols

$$\Gamma_{\mu\nu}^\alpha = \frac{1}{2} g^{\alpha\beta} (\partial_\mu g_{\nu\beta} + \partial_\nu g_{\mu\beta} - \partial_\beta g_{\mu\nu}) \quad (\text{B4})$$

the 19 non-vanishing components are (with the prime indicating the derivative with respect to η):

$$\begin{aligned} \Gamma_{00}^0 &= \Gamma_{11}^0 = \Gamma_{01}^1 = \Gamma_{10}^1 = \Gamma_{02}^2 = \Gamma_{20}^2 = \Gamma_{03}^3 = \Gamma_{30}^3 = \frac{R'}{R} \\ \Gamma_{22}^0 &= \frac{R'}{R} \sin^2 \chi \\ \Gamma_{33}^0 &= \frac{R'}{R} \sin^2 \chi \sin^2 \theta \\ \Gamma_{22}^1 &= -\sin \chi \cos \chi \\ \Gamma_{33}^1 &= -\sin \chi \cos \chi \sin^2 \theta \\ \Gamma_{12}^2 &= \Gamma_{21}^2 = \Gamma_{13}^3 = \Gamma_{31}^3 = \cot \chi \\ \Gamma_{33}^2 &= -\sin \theta \cos \theta \\ \Gamma_{23}^3 &= \Gamma_{32}^3 = \cot \theta \end{aligned}$$

The Ricci tensor

$$\mathcal{R}_\beta^\alpha = g^{\alpha\gamma} (\partial_\delta \Gamma_{\gamma\beta}^\delta - \partial_\beta \Gamma_{\gamma\delta}^\delta + \Gamma_{\gamma\beta}^\delta \Gamma_{\delta\sigma}^\sigma - \Gamma_{\gamma\delta}^\sigma \Gamma_{\beta\sigma}^\delta) \quad (\text{B5})$$

is diagonal:

$$\mathcal{R}_0^0 = -\frac{3}{R^2} \left(\frac{R''}{R} - \frac{R'^2}{R^2} \right) \quad (\text{B6})$$

$$\mathcal{R}_k^k = -\frac{1}{R^2} \left(\frac{R''}{R} + \frac{R'^2}{R^2} + 2 \right) \quad (\text{B7})$$

The Ricci scalar is:

$$\mathcal{R} = \mathcal{R}_\alpha^\alpha = -\frac{6}{R^2} \left(\frac{R''}{R} + 1 \right) \quad (\text{B8})$$

The Einstein tensor

$$G_\nu^\mu = \mathcal{R}_\nu^\mu - \frac{1}{2} \delta_\nu^\mu \mathcal{R} \quad (\text{B9})$$

is diagonal

$$G_0^0 = \frac{3}{R^2} \left(\frac{R'^2}{R^2} + 1 \right) \quad (\text{B10})$$

$$G_k^k = \frac{1}{R^2} \left(2 \frac{R''}{R} - \frac{R'^2}{R^2} + 1 \right) \quad (\text{B11})$$

The case of open universe, $\kappa = -1$, can be obtained by replacing $\sin \chi \rightarrow \sinh \chi$ in the modified RW metric. This amounts to replacing $\chi \rightarrow i\chi$. The previous calculations sail through with the Einstein tensor being generalized to:

$$G_0^0 = \frac{3}{R^2} \left(\frac{R'^2}{R^2} + \kappa \right) \quad (\text{B12})$$

$$G_k^k = \frac{1}{R^2} \left(2 \frac{R''}{R} - \frac{R'^2}{R^2} + \kappa \right) \quad (\text{B13})$$

In the comoving coordinate, from (B2), the 4-vector velocity is: $u_0 = R$, $u_k = 0$ and $u^0 = \frac{1}{R}$, $u^k = 0$. The stress-energy tensor

$$T_\nu^\mu = (p + \epsilon) u^\mu u_\nu - p \delta_\nu^\mu \quad (\text{B14})$$

is diagonal:

$$T_0^0 = \epsilon = \rho c^2 \quad (\text{B15})$$

$$T_k^k = -p \quad (\text{B16})$$

The Einstein field equation

$$G_\nu^\mu = \frac{8\pi G}{c^4} T_\nu^\mu \quad (\text{B17})$$

becomes a set of two independent equations:

$$\frac{3}{R^2} \left(\frac{R'^2}{R^2} + \kappa \right) = \frac{8\pi G}{c^2} \rho \quad (\text{B18})$$

$$\frac{1}{R^2} \left(2 \frac{R''}{R} - \frac{R'^2}{R^2} + \kappa \right) = -\frac{8\pi G}{c^4} p \quad (\text{B19})$$

To convince ourselves about the soundness of our calculations so far, we shall first recover Friedman's equations from the traditional RW metric in what follows.

1. The traditional RW metric and Friedmann's equations

First, note that the traditional RW metric can also be written in the same form as in the modified RW metric above:

$$ds^2 = R^2(\eta) [d\eta^2 - (d\chi^2 + \sin^2 \chi d\Omega^2)]$$

and

$$ds^2 = R^2(\eta) [d\eta^2 - [d\chi^2 + \sinh^2 \chi d\Omega^2]]$$

for $\kappa = \pm 1$ respectively, but with a *different* conformal time (c being a constant):

$$d\eta = c \frac{dt}{R(t)} \quad (\text{B20})$$

Our previous calculations for the Einstein tensor and the Einstein equation are unchanged; namely, Eqs(B18) and (B19) remain valid. To proceed, we need to convert the derivative of R with respect to η into its derivative with respect to t . The conversion is straightforward:

$$\frac{dt}{d\eta} = \frac{1}{c} R \quad (\text{B21})$$

$$R' = \frac{1}{c} R \dot{R} \quad (\text{B22})$$

$$R'' = \frac{1}{c^2} (R \dot{R}^2 + R^2 \ddot{R}) \quad (\text{B23})$$

Equation (B18) reads:

$$\frac{3}{R^2} \left(\frac{1}{c^2} \dot{R}^2 + \kappa \right) = \frac{8\pi G}{c^2} \rho \quad (\text{B24})$$

$$\frac{\dot{R}^2}{R^2} = \frac{8\pi G}{3} \rho - \frac{\kappa c^2}{R^2} \quad (\text{B25})$$

or, in terms of $a = \frac{R}{R_0}$ and $k_0 = \frac{\kappa}{R_0^2}$:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3} \rho - \frac{\kappa c^2}{a^2} \quad (\text{B26})$$

which is the familiar Friedman equation. Eq(B19) reads:

$$\begin{aligned} \frac{1}{R^2} \left(\frac{2}{c^2} \dot{R}^2 + \frac{2}{c^2} R \ddot{R} - \frac{1}{c^2} \dot{R}^2 + \kappa \right) &= -\frac{8\pi G}{c^4} p \\ \frac{\dot{R}^2}{R^2} + 2 \frac{\ddot{R}}{R} + \frac{\kappa c^2}{R^2} &= -\frac{8\pi G}{c^2} p \\ \frac{\ddot{R}}{R} &= -\frac{4\pi G}{3} \left(\rho + 3 \frac{p}{c^2} \right) \end{aligned}$$

or

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left(\rho + 3 \frac{p}{c^2} \right) \quad (\text{B27})$$

which is the familiar Friedman acceleration equation.

2. The modified RW metric and modified Friedmann equations

Let us first recast Eq(B18) (note that the c in the RHS is no longer a constant) as:

$$\frac{3}{R^2} \left(\frac{R'^2}{R^2} + \kappa \right) = \frac{8\pi G}{c^2} \rho = \frac{8\pi G}{c_0^2 R_0} R \rho \quad (\text{B28})$$

$$\frac{R'^2}{R^2} + \kappa = \frac{8\pi G}{3c_0^2 R_0} \rho R^3 \quad (\text{B29})$$

Both sides of this equation are scale invariant since $\rho \propto R^{-3}$. Equation (B19) is recast as:

$$\frac{1}{R^2} \left(2 \frac{R''}{R} - \frac{R'^2}{R^2} + \kappa \right) = -\frac{8\pi G}{c^4} p = -\frac{8\pi G}{c_0^4 R_0^2} R^2 p \quad (\text{B30})$$

$$2 \frac{R''}{R} - \frac{R'^2}{R^2} + \kappa = -\frac{8\pi G}{c_0^4 R_0^2} p R^4 \quad (\text{B31})$$

This equation is also scale invariant since $p \propto R^{-4}$. The conversion between $d\eta$ and dt is:

$$\begin{aligned} \frac{dt}{d\eta} &= \frac{R^{\frac{3}{2}}}{c_0 R_0^{\frac{1}{2}}} \\ R' &= \dot{R} \frac{dt}{d\eta} = \frac{R^{\frac{3}{2}}}{c_0 R_0^{\frac{1}{2}}} \dot{R} \\ R'' &= \frac{d}{d\eta} \left(\frac{R^{\frac{3}{2}}}{c_0 R_0^{\frac{1}{2}}} \dot{R} \right) \\ &= \frac{3}{2} \frac{R^{\frac{1}{2}} R'}{c_0 R_0^{\frac{1}{2}}} \dot{R} + \frac{R^{\frac{3}{2}}}{c_0 R_0^{\frac{1}{2}}} (\dot{R})' \\ &= \frac{3}{2} \frac{R^2}{c_0^2 R_0} \dot{R}^2 + \frac{R^3}{c_0^2 R_0} \ddot{R} \end{aligned}$$

Equation (B29) becomes:

$$\frac{1}{c_0^2 R_0} R \dot{R}^2 + \kappa = \frac{8\pi G}{3c_0^2 R_0} \rho R^3 \quad (\text{B32})$$

$$\frac{\dot{R}^2}{R^2} = \frac{8\pi G}{3} \rho - \frac{\kappa c_0^2 R_0}{R^3} \quad (\text{B33})$$

Equation (B31) becomes:

$$\begin{aligned} 3 \frac{1}{c_0^2 R_0} R \dot{R}^2 + 2 \frac{1}{c_0^2 R_0} R^2 \ddot{R} - \frac{1}{c_0^2 R_0} R \dot{R}^2 + \kappa &= -\frac{8\pi G}{c_0^4 R_0^2} p R^4 \\ \frac{\dot{R}^2}{R^2} + \frac{\ddot{R}}{R} + \frac{\kappa c_0^2 R_0}{2R^3} &= -\frac{4\pi G}{c_0^2 R_0} p R \end{aligned}$$

or

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3} \left(2\rho + 3 \frac{pR}{c_0^2 R_0} \right) + \frac{\kappa c_0^2 R_0}{2R^3} \quad (\text{B34})$$

In terms of $a \equiv \frac{R}{R_0}$ and $k \equiv \frac{\kappa}{R_0^2}$, this concludes our derivation of the new equations that govern the evolution of the scale factor:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho - \frac{kc_0^2}{a^3} \quad (\text{B35})$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(2\rho + 3\frac{p}{c_0^2}a\right) + \frac{kc_0^2}{2a^3} \quad (\text{B36})$$

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